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Journal

of

THE ROYAL AERONAUTICAL SOCIETY

NOVEMBER 1960

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GRADUATES' AND STUDENTS' SECTION

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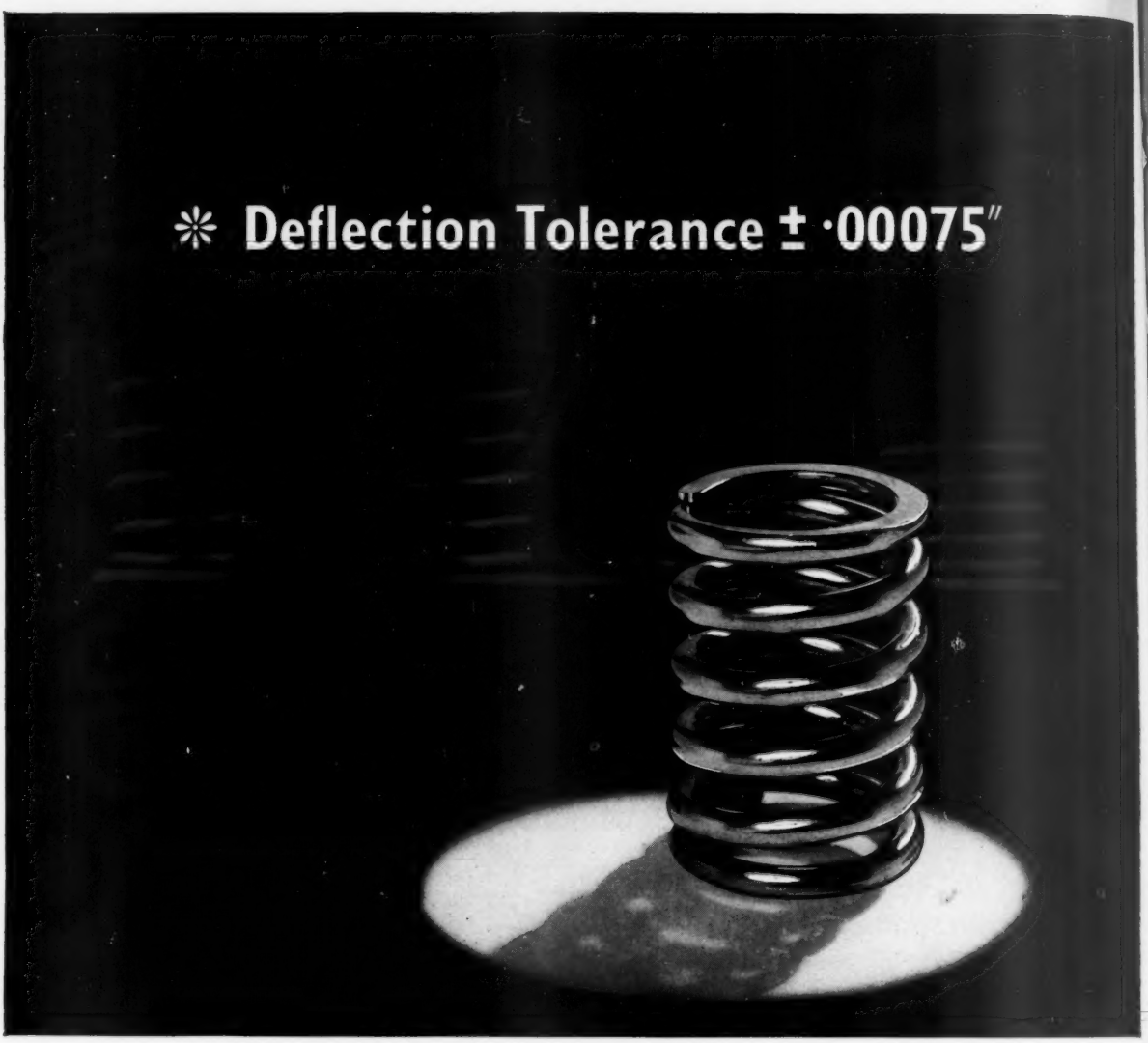

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Journal of
THE ROYAL AERONAUTICAL SOCIETY

INCORPORATING THE INSTITUTION OF AERONAUTICAL ENGINEERS AND THE HELICOPTER ASSOCIATION OF GREAT BRITAIN

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All communications for publication in the JOURNAL *should be addressed to the Editor,* JOAN BRADBROOKE, *A.R.Ae.S.; all communications on general matters affecting the Society should be addressed to the Secretary,* A. M. BALLANTYNE, *T.D., B.Sc., Ph.D., F.R.Ae.S., Hon.F.C.A.I., F.I.A.S.*

None of the papers or paragraphs must be taken as expressing the opinion of the Council of the Royal Aeronautical Society unless otherwise stated.

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(Founded 1866)

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NOTE: Under the revised By-Laws, allowed on 11th January 1960, the President of each Division and the Chairman of each Section of the Society is a Member of Council by reason of his office.

NOTICES

Christmas Holiday

The Offices and Library of the Society will be closed from Friday afternoon 23rd December, until 9 a.m. on Wednesday 28th December.

Associate Fellowship Examination—December 1960

The next Associate Fellowship Examination will be held on Tuesday, Wednesday, Thursday and Friday, the 13th, 14th, 15th and 16th December. All candidates will be sent a detailed timetable.

LECTURE THEATRE APPEAL

The following lists show further donations received up to the time of going to press.

Donations

	£ s. d.	£ s. d.
Previous total (after minor adjustments)		£15,633 11 9
Dr. D. Cameron	3 3 0	
Canadian Aeronautical Institute	30 0 0	
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7-year Covenants

Previous total (after minor adjustments)	*£5,436 0 4

With Income Tax at the present standard rate of 7s. 9d. in the £ this figure should produce* **£62,126 *(approx.) over the 7-year period.*

10-year Covenants

Previous total	*£1,612 10 0

With Income Tax at the present standard rate of 7s. 9d. in the £ this figure should produce* **£26,325 *(approx.) over the 10-year period.*

Annual Payments (unspecified period)

Previous total	£87 10 0
D. J. Gorman, Esq.	2 2 0
	£89 12 0

Further promises amounting to **£470** *have been received and donations in kind promised by firms are valued at* **£3,500** *(approx.).*

GRAND TOTAL (approximately) ... £108,646

The following have also subscribed but do not wish the amounts of their contributions to be published:—

Mrs. H. G. Alston K. F. Venn, Esq.

N. E. Rowe Medal Competition

Branch Secretaries are reminded that the closing date for submission of papers for the 1961 N. E. Rowe Medals is **31st December 1960.**

Lecture Theatre—Opening Ceremony

As members will have learned from the leaflet enclosed with the October Journal, the Opening Ceremony of the Lecture Theatre will be **on Friday 2nd December.**

Lord Brabazon of Tara, P.C., G.B.E., M.C., Hon. F.R.Ae.S., a Past-President of the Society, will officially open the Lecture Theatre at 7 p.m. There will be a short Presidential Address by Dr. E. S. Moult, C.B.E., B.Sc., F.R.Ae.S.

Members are reminded that an application form for tickets for them and their ladies—25s. each including buffet supper—was enclosed with the **September Journal.** This form should be used for the new date and those who have not yet applied for tickets are advised to do as quickly as possible. The closing date for application is **30th November.**

The Library

The main library is again open to members. It will be some time, however, before the stock of back numbers of some periodicals can be sorted and this side of the library service will be adversely affected. Reports and preprints abstracted or listed in the Journal will continue to be available for loan.

The New By-Laws

With this issue of the Journal members will receive a copy of the new By-Laws. Members' attention is particularly drawn to By-Law 2(D) governing Fellowship.

Symposium on Supersonic Transport Aircraft

The Symposium will be held in the Lecture Theatre at 4 Hamilton Place on 8th December 1960 from 9.30 a.m. to approximately 5.30 p.m. The subjects will cover: Aerodynamics, Structures, Materials, Propulsion, Airworthiness, Piloting Problems, Systems Engineering and Operation and Economics. Each subject will be followed by a discussion and a general discussion will be held at the end. **Admission will be by ticket only.** Programmes, containing summaries of each subject, and tickets may be obtained from the Secretary, Royal Aeronautical Society, 4 Hamilton Place, London, W.1.

Symposium on Electromagnetics and Fluid Dynamics of Gaseous Plasma

The eleventh annual international Polytechnic Institute of Brooklyn Symposium on Electromagnetics and Fluid Dynamics of Gaseous Plasma will be held from 4th to 6th April 1961 in New York City.

The programme will include some contributed papers. The closing date for submission of papers or 200 word abstracts is 20th December 1960. Full details may be obtained from Professors Martin Bloom and Enrico Levi, Symposium Committee, Polytechnic Institute of Brooklyn, 55 Johnson Street, Brooklyn 1, New York.

Univac Computer Courses

A series of Univac Computer Courses is to be held by Remington Rand Ltd., to provide information on programming and application of Univac Computing Systems.

Courses will last for one week and be given at six weekly intervals beginning on 14th November 1960. Full details may be obtained from Remington Rand Ltd., Univac Computer Division, Remington House, 61/65 Holborn Viaduct, E.C.1.

DIARY

MAIN SOCIETY

All lectures in London will be held in the Lecture Theatre, 4 Hamilton Place, unless otherwise stated.

11th November
Joint Lecture with the Institution of Electrical Engineers. The Future of Electrics and Electronics in Aircraft and Guided Missiles. The Rt. Hon. The Viscount Caldecote. **Savoy Place, Victoria Embankment, W.C.2. 5.30 p.m.** (Tea at 5 p.m.)

18th November
SIXTEENTH BRITISH COMMONWEALTH LECTURE—The Story of Indian Air Transport. J. R. D. Tata. Institution of Mechanical Engineers, Birdcage Walk, S.W.1. **6 p.m.** (Tea at 5.30 p.m.).

21st November
HISTORICAL GROUP—My First Ten Years in Aviation. Sir Thomas Sopwith. Institution of Mechanical Engineers, Birdcage Walk, S.W.1. **6 p.m.** (Tea at 5.30 p.m.)

22nd November
Basic Aerodynamic Considerations of V/STOL Aircraft. P. L. Sutcliffe. Library, 4 Hamilton Place. **7 p.m.**

28th November
AGRICULTURAL AVIATION GROUP—The Farmer's Viewpoint. J. B. Farrant. Library, 4 Hamilton Place. **7 p.m.**

30th November
MAN POWERED AIRCRAFT GROUP—Some Aspects of Bird Flight. John Barlee. Library, 4 Hamilton Place, **7 p.m.**

1st December
MAIN LECTURE AT LUTON BRANCH—Ballistic Research Rockets, with Particular Reference to Black Knight. D. J. Lyons. Assembly Hall, Luton High School, Alexandra Avenue, Luton. **6.30 p.m.**

2nd December
LECTURE THEATRE—OPENING CEREMONY. 7 p.m. Admission by ticket only.

7th December
AGRICULTURAL AVIATION GROUP—Grass, Soils and Fertilisers. I. A. Nicholson. **7 p.m.**

8th December
SYMPOSIUM—Supersonic Transport Aircraft. **9.30 a.m.**

9th December
MAN POWERED AIRCRAFT GROUP—Aerodynamic Problems of Man Powered Flight. T. R. F. Nonweiler. **7 p.m.**

12th December
HISTORICAL GROUP—Some Problems of Aeronautical History. C. H. Gibbs-Smith. **7 p.m.**

15th December
Economic Lessons from Short-haul Airline Operations. S. F. Wheatcroft. **6 p.m.** (Tea at 5.30 p.m.)

ASTRONAUTICS AND GUIDED FLIGHT SECTION

6th December
Solid Propellant Rocket Motors. Dr. G. H. S. Young and Dr. W. R. Maxwell. **6 p.m.**

GRADUATES' AND STUDENTS' SECTION

23rd November
The British Space Programme. M. O. Robins. Library, 4 Hamilton Place. **7.30 p.m.**

25th November
Annual Dance. Library, 4 Hamilton Place. **8 p.m.**

14th December
B.O.A.C. and the Future. Charles Abell. **7.30 p.m.**

ROTORCRAFT SECTION

29th November
The Design of Ground Effect Machines. R. Stanton Jones. Library, 4 Hamilton Place. **6 p.m.**

BRANCHES

14th November
Henlow—Accident Investigation. P. G. Tweedie. Assembly Hall, R.A.F. Technical College. 7.45 p.m.

15th November
Luton—Certification of Aircraft in the U.S.A. K. H. Greenly. Napier Senior Staff Canteen, Luton Airport. 6 p.m.

16th November
Christchurch—The Design of the Bluebird Hydroplane. L. H. Norris and K. H. Norris. King's Arms Hotel. 7.30 p.m.
Coventry—Flight Development of Modern Production Aircraft. C. F. Bethwaite. Herbert Art Gallery. 7.30 p.m.
Manchester—Black Knight. H. G. R. Robinson. Reynolds Hall, College of Technology. 7.30 p.m.
Southampton—Eighth Mitchell Memorial Lecture. Man Powered Aircraft. B. S. Shenstone. Engineering Lecture Theatre, University of Southampton. 8 p.m.
Southend—Film Show (including "Antarctic Crossing"). Room 3, Engineering Dept., Municipal College. 7.30 p.m.

17th November
Bristol—Helicopters. Conference Room, Bristol Helicopters Ltd., Weston-super-Mare. 7 p.m.
Cheltenham—Air Traffic Control. W. C. Woodruff. St. Mary's College, St. George's Place, Cheltenham. 7.30 p.m.

23rd November
Glasgow—Graduates and Students—Lecture by T. H. Cain. Engineering Building, University of Glasgow. 7.30 p.m.
Weybridge—Joint Meeting with the Southern Branch of the Institution of Mechanical Engineers. The Art of Developing Aero Engines. A. C. Lovesey. Apprentice Training School, Vickers-Armstrongs (Aircraft) Ltd. 6 p.m.

24th November
Bristol—Junior Members' Paper Competition. Conference Room, Filton House, Bristol Aircraft Ltd. 6 p.m.

28th November
Boscombe Down—Hovercraft. G. Elsely. Lecture Hall, A. & A.E.E. 5.30 p.m.
Henlow—Presidential Address. Air Commodore N. C. S. Rutter. Assembly Hall, R.A.F. Technical College, Henlow. 7.45 p.m.

1st December
Belfast—Accident Investigation. G. W. Acock.
Luton—MAIN SOCIETY LECTURE—Ballistic Research Rockets with Particular Reference to Black Knight. D. J. Lyons. Assembly Hall, Luton High School, Alexandra Avenue. 6.30 p.m. Followed by Annual Dinner, Biscot Mill Hotel.

5th December
Derby—The de Havilland 121. C. T. Wilkins. Rolls-Royce Welfare Hall, Nightingale Road. 6.15 p.m.

7th December
Bristol—Joint Meeting with the Institute of Production Engineers. Main Lecture Theatre, University Engineering Labs., University of Bristol. 7 p.m.
Brough—Some Aeronautical Applications of Computers. Dr. S. H. Hollingdale. Royal Station Hotel, Hull. 7.30 p.m.
Christchurch—Film Show. King's Arms Hotel. 7.30 p.m.
London Airport—The History of Silver City Car Ferry Operations. Michael Day. Senior Mess, B.O.A.C. H.Q. 6 p.m.
Swindon—Aircraft in the Modern Army. Brig. R. A. Fyffe. The College, Victoria Road. 7.30 p.m.
Weybridge—Flying with the Trans-Antarctic Expedition. Wing Cdr. J. H. Lewis. Apprentice Training School, Vickers-Armstrongs (Aircraft) Ltd. 6 p.m.

8th December
Isle of Wight—Man Powered Flight. B. S. Shenstone. Saunders-Roe Clubhouse. 6 p.m.
Southampton—Young People's Lecture. T. R. F. Nonweiler. Engineering Lecture Theatre, University of Southampton. 8 p.m.

12th December
Henlow—The History and Development of Martin-Baker Ejection Seats. Wing Cdr. J. Jewell. Assembly Hall, R.A.F. Technical College. 7.45 p.m.

13th December
Gloucester—Helicopter Operations. Capt. R. W. Dibb. Wheatstone Hall, Brunswick Road, Gloucester. 7.30 p.m.

14th December
Manchester—Nuclear Propulsion of Space Flight. Dr. L. R. Shepherd. Reynolds Hall, College of Technology. 7.30 p.m.
Southend—"Hovercraft," with films. J. W. Lloyd. Room 3, Engineering Dept., Municipal College, London Road. 7.30 p.m.

15th December
Bristol—Rocket Motors. S. Allen. Conference Room, Filton House, Bristol Aircraft Ltd. 6 p.m.
Coventry—Junior Lectures and Film Show. Herbert Art Gallery. 7.30 p.m.
Glasgow—Graduates and Students—Light Aircraft in General. Scottish Aviation Ltd., Prestwick. 7.30 p.m.

16th December
Weybridge—Annual Dance.

Elections

The following is a list of elections and transfers of membership:—

Associate Fellows

Roy Alexander
Eric Auty
John Darcy Baker-Carr
John Stanley Bell (*from Graduate*)
Clifford Lester Bore (*from Graduate*)
Reginald William Brecknock
Alistair Rhodes Campbell
Alistair Peter Wishart Cane (*from Associate*)
Gervaise Maurice Geoffrey Cooper (*from Graduate*)
Henry George Cuming
Alan Russell Dare (*from Graduate*)
Ian Davidson (*from Graduate*)
Ronald Walter Dibb
George Dillaway
Derek Lloyd Fairweather (*from Graduate*)
Alfred Henry Fox
Tom Frisby Goodwin (*from Graduate*)
William Gordon
Edward Gregory (*from Associate*)
Edward Vernon Griffith (*from Graduate*)
John Ashby Guy
Raymond Alan Harrison (*from Graduate*)
Donald Bryan Hatfield
James Francis Hawthorne
Robert John Hayter
Alberto Jona
Kenneth Lewis
Desmond Arthur John Lodge
Arthur Glenville Longden
Donald John McAllister
Keith Alexander MacKenzie (*from Associate*)
James Cleland McVeigh (*from Graduate*)
Arthur Cambridge Merrington
George Arthur Grindell Morris (*from Associate*)
Edward J. Nesbitt
Dhanvada Madhava Rao (*from Graduate*)
Peter John Reen
John Francis Millar Scholes
Arthur William Skingsley
W. Thomas Stark
Laurence Rudolf Trier (*from Associate*)
Noel Alfred Vear
Joseph Venn
Geoffrey Elliott Williams
Sidney Richard Williams
Harold Wood

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Fateh Singh Bhinder
John Cameron
Donald Campbell
Michael Chandler
Anthony John Clarke
Gerald Malcolm Dickson
David Wesley Griffiths
Herbert George Hart
Colin David Stuart Kennedy
William Oscar Lee
Frederick Ronald Loveday
Charles Howard Matthew Maddox
Frank Conwy Morgan
John Ross Muddell
John Edwin Nunn
Robert William Goodfellow Ruark
James William Sewell
Alfred John Sliney
William Eustace Stanton
Leslie John Prickett Swabey
Herbert Wilkinson
Walter Rice Williams
John Peter Wilson

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Deane Robert Blackman (*from Student*)
Nicholas Cadoux-Hudson
Alan David Futter
Graeme Bernard Gilmore
James Cyril Groom
John Brian Henshaw (*from Student*)
Douglas William Laurie-Lean (*from Student*)
Thomas Martyn Anthony Lewis
David John Macgeorge
Timothy James Morgan
Richard Murray
Michael Alfred Peace
George Graham Readett (*from Student*)
Peter Richard Reddaway
Geoffrey Charles Roberts
Michael Ross
John Charles Philip Stott (*from Student*)
Geoffrey John Sturgess (*from Student*)
Arthur Gilbert Ramsay Thomson (*from Student*)
John Bryan Wilby
Quentin Wilson (*from Student*)

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Michael Norman Baker
John Dennis Coleman
Douglas John Cox
Christopher Hendrik Friend
Brian Keeley
Paul Francis Kilty
John Allen McLean
Michael Anthony Peter McVeigh
Michael Peascod
Robert William Ernest Shannon
William Graham Wells

Companions

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A. W. Kamfiord
Grenville Gordon Oliver Manton
Raymond Horace Smith
Brian Thomas Sweeney

Annual Subscriptions

Members are reminded that their annual subscriptions become due on 1st January 1961. The rates are:—

	Home £ s. d.	*Abroad* £ s. d.
Fellows	8 0 0	7 0 0
Fellows (over 65)	5 0 0	4 0 0
Associate Fellows	6 0 0	5 0 0
Associate Fellows (over 65)	4 0 0	3 0 0
*Associates	5 0 0	5 0 0
Associates (over 65)	3 0 0	3 0 0
Graduates (aged under 26)	2 10 0	2 10 0
Graduates (aged 26 and over)	3 0 0	3 0 0
Students (aged under 21)	1 10 0	1 10 0
Students (aged 21 and over)	2 0 0	2 0 0
Companions	4 0 0	4 0 0
Companions (over 65)	3 0 0	3 0 0
Companions (No Journal)	2 10 0	2 10 0

*Any Associate elected before 1st October 1947 may, if he wishes elect not to receive the Journal; in this case his subscription will be reduced by £1 10s. 0d. to £3 10s. 0d.

It will avoid delay and confusion if members sending remittances for subscriptions, will state their names, addresses and grades of membership clearly. Remittances should be made payable to the Royal Aeronautical Society.

Copy of Letter from Chief Inspector of Taxes Branch, Inland Revenue, Dated 17th October 1958

Reference: C.I./SUB/306

I have to inform you that the Commissioners of Inland Revenue have approved The Royal Aeronautical Society for the purposes of Section 16, Finance Act, 1958, and that the whole of the annual subscription paid by a member who qualifies for relief under that Section will be allowable as a deduction from his emoluments assessable to income tax under Schedule E. If any material relevant change in the circumstances of the society should occur in the future you are requested to notify this office.

I should be glad if you would inform your members as soon as possible of the approval of the society. The circumstances and manner in which they may make claims to income tax relief are described in the following paragraphs, the substance of which you may care to pass on to your members.

Commencing with the year to 5th April 1959, a member who is an office holder or employee is entitled to a deduction from the amount of his emoluments assessable to income tax under Schedule E of the whole of his annual subscription to the society *provided that*—

(*a*) the subscription is defrayed out of the emoluments of the office or employment, *and*,

(*b*) the activities of the society so far as they are directed to all or any of the following objects—

(*i*) the advancement or spreading of knowledge (whether generally or among persons belonging to the same or similar professions or occupying the same or similar position);

(*ii*) the maintenance or improvement of standards of conduct and competence among the members of any profession;

(*iii*) the indemnification or protection of members of any profession against claims in respect of liabilities incurred by them in the exercise of their profession:

are relevant to the office or employment, that is to say, the performance of the duties of the office or employment is directly affected by the knowledge concerned or involves the exercise of the profession concerned.

A member of the society who is entitled to the relief should apply to his tax office as soon as possible *after* 31*st October* 1958, for form P358 on which to make a claim for adjustment of his pay as you earn coding.

(*signed*) T. Dunsmore,
Senior Principal Inspector of Taxes.

News of Members

B. W. Anderson (*Student*) has completed his Apprenticeship with the de Havilland Aircraft Co. Ltd., and is now a Stressman, English Electric Aviation Ltd., Warton.

R. D. Anthony (*Associate Fellow*) formerly with the Nuclear Power Group is now with the Nuclear Inspectorate of the Ministry of Power.

Air Commodore F. R. Banks (*Fellow*) has been appointed Managing Director of Blackburn Engines Ltd.

N. W. Barford (*Associate Fellow*) formerly Senior Design Engineer, Lockheed Aircraft Corp., Marietta, is now a Senior Engineer, Vertol Division of Boeing Airplane Co., Morton.

A. Bialkowski (*Student*) formerly Senior Draughtsman, English Electric Aviation Ltd., Acton, is now Design Draughtsman, Blackburn Aircraft Ltd., London.

Wing Cdr. E. J. Brice (*Associate Fellow*) formerly in Command No. 1 Parachute Training School, Abingdon, is now at the R.A.F. College, Cranwell.

B. J. Brinkworth (*Associate Fellow*) formerly at the R.A.E. Farnborough is now a Lecturer in Mechanical Engineering at the University College of South Wales and Monmouthshire.

D. Brown (*Associate Fellow*) formerly Stress Engineer, North American Aviation, Columbus, Ohio, is now with the Space Task Group of N.A.S.A. at Langley Field.

Sqn. Ldr. G. R. D. Calder (*Associate Fellow*) formerly a Student at the R.A.F. Staff College, Andover, is now a Lecturer in Propulsion at the R.A.F. Technical College, Henlow.

Air Vice-Marshal B. A. Chacksfield (*Associate Fellow*) formerly Senior Air Staff Officer, R.A.F. Brampton, has now assumed Command of Headquarters No. 22 Group, R.A.F. Market Drayton.

Wing Cdr. J. Crane (*Associate Fellow*) formerly of the Guided Weapons Department, Royal Radar Establishment, Malvern, is now Wing Commander, Technical Trials at R.A.F. North Coates.

Sqn. Ldr. K. B. Crosby (*Associate*) formerly of R.A.F. Changi, is now at Headquarters United Kingdom Air Traffic Service, R.A.F. Bentley Priory, as Plans I.

Group Capt. D. R. Cuming (*Associate*) formerly Officer Commanding R.A.A.F. Edinburgh, Australia, is now Staff Officer Aircraft Engineering, Support Command H.Q. R.A.A.F. Melbourne.

Sqn. Ldr. E. T. Curran (*Associate Fellow*) formerly at the R.A.F. Technical College, Henlow, is now at the Wright-Patterson A.F. Base, Dayton, Ohio.

R. W. Edwards (*Companion*) is now a Design Development Engineer for Wake Engineering Ltd., Brighton.

P. T. Fink (*Associate Fellow*) has been appointed the Peter Nicol Russell Professor of Mechanical Engineering in the University of Sydney.

Lt.-Col. R. C. Fletcher (*Associate Fellow*) formerly Senior Technical Officer of Tegwra has been appointed Assistant Director (Electronics) on promotion in the new Directorate of Communications & Electronics, War Office.

A. Frenkel (*Associate Fellow*) formerly Senior Designer, Bristol Aircraft (Western) Ltd., Stevenson Field, is now with Bristol Aero-Industries Ltd., Montreal.

R. A. Fuller (*Associate Fellow*) formerly a gas turbine technician, Bristol Siddeley Engines Ltd., is now a Test Engineer, Engineering Dept., Massey Ferguson Ltd., Coventry.

V. G. Gadgil (*Associate Fellow*) formerly Engineering Manager, Air-India International, is now Controller of Stores and Purchases.

Lt.-Col. P. Garnell (*Associate Fellow*) formerly War Office (E.2), is now on the lecturing staff of the Royal Military College of Science, Shrivenham.

C. K. Giles (*Associate*) formerly Assistant Publicity Manager, Vickers Ltd., Westminster, is now Publicity Manager for Simon Engineering Ltd., Stockport.

J. G. Gill (*Associate Fellow*) formerly Director and Vice-President of Operations, A. V. Roe Group of Canada, is now Assistant Director of Process Development, Crucible Steel Co. of America, Pittsburgh.

W. J. Gornall (*Associate Fellow*) formerly Acting Chief Stress Engineer, Government Aircraft Factories, Melbourne, is now Design Engineer (Structures), Commonwealth Aircraft Corp., Melbourne.

H. Greene (*Associate Fellow*) formerly Designer, Aircraft, Grumman Aircraft Engineering Corp., New York, is now in the Stress Department, Short Brothers and Harland Ltd., Belfast.

R. S. Hammond (*Associate Fellow*) formerly a Senior Stressman, de Havilland Aircraft Co., Airspeed division, Christchurch, is now Senior Engineer, Projects Group, English Electric Aviation Ltd., Warton.

Flying Officer J. S. Haslam (*Graduate*) formerly Quality Engineer, Rolls-Royce Ltd., Glasgow, is now Flying Wing Engineer, Lindholme.

Flt. Lt. J. W. J. Hawkins (*Graduate*) formerly Technical Officer, R.A.F. Butzweilerhof, has been posted to the R.A.F. Technical College, Henlow, for post-graduate studies.

B. D. Henshall (*Associate Fellow*) formerly Head of Special Projects, Weapons Research Division, A. V. Roe and Company Ltd., is now Senior Staff Scientist with AVCO-Research and Advanced Development Division, Wilmington.

M. F. Higginson (*Associate*) has left the R.A.F. and is now a Design Draughtsman, Auster Aircraft Ltd.

C. D. S. Hogg (*Associate*) formerly Examiner A.I.S. at R.A.F. Lyneham is assuming similar duties at A.R.S. Seletar.

Ed. H. Holder (*Associate Fellow*) has returned to the United Kingdom from secondment to Pakistan Ordnance Factories at Wah Cantt. W. Pakistan and is now Principal Production Officer, D.A.W. (P), Ministry of Aviation.

B. G. Hunt (*Student*) formerly with A. V. Roe & Co. Ltd., Manchester, is now with the Australian Department of Supply, Trials and Instrumentation Wing, W.R.E., Salisbury.

K. E. Johnson (*Associate Fellow*) formerly with the Production Development Department, Vickers-Armstrongs (Aircraft) Ltd., is now an Engineer III, Ministry of Aviation.

Sqn. Ldr. A. H. Jones (*Associate*) formerly Technical Services, Air Ministry, is now Technical Co-ord/Plans at H.Q. Fighter Command, Bentley Priory.

Sqn. Ldr. E. T. F. Jones (*Associate*) formerly R.A.F. Changi, is now Training 1, Headquarters Maintenance Command, R.A.F. Amport.

J. C. King (*Associate Fellow*) formerly Assistant Director of Engineering, English Electric Co. Ltd., is now Manager of the Company's Aircraft Equipment Division.

Air Vice-Marshal E. Knowles (*Associate Fellow*) formerly at Headquarters Technical Training Command, is now at the Air Ministry as Director of Educational Services.

P. W. Lambell (*Associate Fellow*) formerly A.I.D. Inspector in charge of London No. 1 District Office, is now with Technical Services, Shell Mex House.

D. R. Leonard (*Graduate*) formerly Section Leader, Aeroelastic Section, Blackburn Aircraft Ltd., is now with the Dynamics Section, Vickers-Armstrongs (Aircraft) Ltd.

G. J. Lewis (*Associate Fellow*) formerly with D. Napier & Son Ltd., is now Assistant Project Engineer, Palmer Aero Products Ltd.

J. Lewis (*Associate Fellow*) formerly Lecturer, Bristol College of Technology, is now Head of the Engineering and Building Department at the College of Further Education, Oxford.

R. P. Llewelyn (*Graduate*) formerly Aero Engine Instructor, Imperial Ethiopian Air Force, Arabia, is now a Performance Engineer, Sir W. G. Armstrong Whitworth Aircraft Ltd., Whitney.

J. H. London (*Associate Fellow*) formerly Structures Engineer, Lockheed Missile & Space Division, Hatfield, is now in the Technical Office, de Havilland Aircraft Co.

N. L. LUPTON (*Associate Fellow*) formerly with Microcell Ltd., is now Contracts Manager, Electronic and Equipment Group, Plessey Company Ltd.

W. MCCAFFREY (*Student*) formerly in Napier's Drawing Office is now a Resident A.I.D. Examiner, Lockheed Ltd., Liverpool.

A. S. MCCLYMONT (*Student*) formerly serving an apprenticeship with Scottish Aviation Ltd., Prestwick, is now a Technician, Aircraft Research Division, A. V. Roe & Co. Ltd., Woodford.

Sqn. Ldr. J. W. MILLER (*Associate Fellow*) formerly with the Directorate of Weapons Engineering, Air Ministry, is now Group Armament Officer, H.Q. No. 12 Group.

Group Capt. C. T. NANCE (*Associate Fellow*) formerly Technical Member of Directory Staff, R.A.F. Staff College, Andover, is now Command Engineering Officer, Far East Air Force, Singapore.

Sqn. Ldr. A. E. PEART (*Associate Fellow*) has been posted from Eng. 2 R.A.F. Transport Command to the Ministry of Aviation on appointment as D.D.P.O./A.C.2.

B. J. POWELL (*Graduate*) formerly in the Aerodynamics Department, Fairey Aviation Ltd., is now an Aerodynamicist in the Advanced Project Group, Hawker Siddeley Aviation Ltd.

L. W. S. RICHARDS (*Associate Fellow*) formerly Maintenance Design Engineer, Bristol Aircraft Co. Ltd., is now Stress Engineer, Vickers-Armstrongs (Aircraft) Ltd.

L. W. ROSENTHAL (*Associate Fellow*), formerly Manager, SARO Structures Ltd., E. Cowes, is now Chief Hovercraft Engineer, Folland Aircraft Ltd., Hamble.

B. S. SHENSTONE (*Fellow*), Chief Engineer of B.E.A., has been appointed to the Board.

P. G. SIMPKINS (*Graduate*) having completed his course at California Institute of Technology, is now a Research Assistant, Hypersonics Group, Imperial College of Science and Technology.

G. T. SMITH (*Associate Fellow*) formerly Regional Sales Manager, Bristol Siddeley Engines Ltd. is now Sales Manager of the Company's Aero-Engine Division.

T. J. STICKINGS (*Student*) formerly Technician, Guided Weapon Development, R.A.F. (with A. V. Roe & Co. Ltd.), is now at W.R.E., Salisbury.

G. J. STURGESS (*Student*) formerly a Student at Loughborough College is now Technical Engineer, Performance and Power Plant Engineering Department, Bristol Siddeley Engines Ltd.

G. H. TIDBURY (*Associate Fellow*), formerly a Project Engineer (Development), Vauxhall Motors Ltd., has been appointed Lecturer at the Advanced School of Automobile Engineering, Cranfield.

FARADAY LECTURE, 1960/61

The Institution of Electrical Engineers' Faraday Lecture for 1960/61 is to be given by L. J. Davies, C.B.E., on "Transistors and All That." The lecture will be given in Birmingham, Bristol, Edinburgh, Leeds, Leicester, London, Manchester, Newcastle, Portsmouth, Rugby and Swansea between 16th November 1960 and 23rd March 1961.

Full particulars may be obtained from the Secretary, Institution of Electrical Engineers, Savoy Place, London, W.C.2.

CHANGES OF ADDRESS OR APPOINTMENT

To assist in keeping the records of members correct and up to date the Secretary will be glad if all members will notify him as soon as possible of changes of address. He would also like to know of any change of appointment.

When notifying changes please give the following particulars:—

Name (in block letters) Grade of Membership
New Address (in block letters) Old address

New appointment.—Please give name and address of employer and position held.

Changes of address must be received *before the 15th of the month* in order to be effective for the following month's JOURNAL.

SOCIETY OF ENVIRONMENTAL ENGINEERS

The Society of Environmental Engineers, formed just over a year ago, will hold lectures and meetings in London, Bristol, Coventry, Manchester, Edinburgh and Glasgow during the 1960-61 session. The next lecture, "Vibration testing and instrumentation of large stores" will be held on 23rd November at Imperial College at 6 p.m.

Full details of the lecture programme and application forms for membership may be obtained from the Secretary, Society of Environmental Engineers, 42 Manchester Street, London W.1.

Membership is open to Scientists and Engineers working in the field of environmental engineering.

JOURNAL BINDING

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Journals, with a note of the name and address of the sender, should be sent direct to The Lewes Press, Friars Walk, Lewes, Sussex, and the remittance to the Secretary at the Offices of the Society.

Members are asked to be certain that the address to which they want their Journals sent is the same on their letters to The Lewes Press and to the Society.

Lecture Summaries

BASIC CONSIDERATIONS OF V/STOL AIRCRAFT

P. L. SUTCLIFFE, B.Sc., D.N.P.I., A.F.R.Ae.S., A.F.I.A.S.

To be given on 22nd November 1960

The main theme is a discussion of the broad aerodynamic and propulsive aspects of aircraft having airfield performances within the meaning of V/STOL. To facilitate this a proposal is made for a universal classification of airfield distance which would remove much of the ambiguity of many comparisons now made under the general heading of STOL.

The lecture then discusses the evaluation of the thrust required to fly at very low forward speeds and the possibility of minimising this by using thrust deflection. The airfield performance is then shown to be uniquely related to the minimum level speed capability of the aircraft, which leads to a simple classification of the aircraft characteristics required for various degrees of V/STOL.

Problems of controllability of V/STOL aircraft are discussed and the influence of control requirements on the installed thrust/weight ratio of VTOL aircraft emphasised. The penalties of demanding full safety factors at maximum weight in hot and high conditions is also covered.

The general analysis is then used to correlate aircraft configuration with airfield performance and to indicate that there appear to be reasonably distinct lines of demarcation between various types of V/STOL aircraft, depending on the particular degree of airfield performance required.

Graduates' and Students' Section

THE BRITISH SPACE RESEARCH PROGRAMME

M. O. ROBINS

To be given on 23rd November 1960

Space research is research carried out by using a particular technique, that of rockets, to carry instruments into regions of the upper atmosphere or beyond. The payload may have the short life given by a vertical sounding rocket, or the long life given by a satellite. Each has its own particular values for research purposes, and these are mentioned.

The British programme of upper atmosphere research with rockets has been in progress for several years, using the research rocket Skylark. This is briefly described, with some of the experimental programme at present in being or contemplated.

Results of great interest can be achieved by a study of optical and radio ground based observations of satellites. British workers have already made substantial advances in this field, some of which are described.

Arrangements have been made with the U.S.A. authorities that British instruments should be installed in American launched Scout satellites. Some of the British plans for this joint programme are described, with some of the aims it is hoped will be achieved.

The capabilities of British rocket designs for launching earth satellites are briefly mentioned, together with some of the scientific researches that could be undertaken with such satellites.

Agricultural Aviation Group

THE FARMER'S VIEWPOINT

J. B. FARRANT

To be given on 28th November 1960

The lecturer will outline the present practice of fertiliser application and crop spraying together with the cost of carrying out the programme by each farmer with his own equipment. He will then discuss whether the farmer can change his programme of application and type of fertiliser used to enable aerial application to take over the job and will show the saving in labour and machinery costs if a change is made. Also, he will point out the necessity for a thorough programming of supply, storage and priority of some applications.

Finally he will give a brief comment on his personal view: if British farming is to compete adequately with competition from cheap labour countries, aerial application of fertilisers and crop sprays may well be part of the answer.

Rotorcraft Section

THE DESIGN OF GROUND EFFECT MACHINES

R. STANTON JONES

To be given on 29th November 1960

The time is rapidly approaching when the ground effect machine is emerging from its crude experimental stage to the point where it will soon be making a serious bid to be considered for a place in the transport field.

Already the refreshingly simple "back-yard" approach, which is delighted with anything that works, is giving way to the professional engineering concern of making these craft work reliably, economically and safely.

This paper attempts to give a general indication of the major design and engineering problems that face the hovercraft designer. The paper starts with a description of the design procedure by which an optimum hovercraft design is approached, and is based on the results of several detailed design studies, which have led to much more realistic estimates of the structural and transmission weights of these vehicles.

The main conclusions are that although it was once thought that very large hovercraft would be necessary before they could become economically competitive, it would now appear that possibly considerably smaller machines may achieve this.

The two other problems which are discussed in some detail are the important ones of dynamic stability and of control. Both these factors influence the power/weight ratios and hence the economics of the hovercraft. It is shown how the dynamic stability is related to the over-wave performance and its effect on the impact and structural design criteria.

Man Powered Aircraft Group

SOME ASPECTS OF BIRD FLIGHT

JOHN BARLEE

To be given on 30th November 1960

The lecturer intends to consider one or two groups which show a well-marked gradation in flight ability which can be related to their way of life. In particular ducks and other water-birds; birds of prey, sea-birds. He hopes to show 50-100 slides of birds in flight.

MAIN LECTURE AT LUTON BRANCH

BALLISTIC RESEARCH ROCKETS—WITH PARTICULAR REFERENCE TO THE BLACK KNIGHT ROCKET

D. J. LYONS

To be given on 1st December 1960

First the relative merits of various types of ballistic test rockets for upper air exploration are considered. Then the main research problems of the design of ballistic rockets are outlined and the factors relevant to the choice of type related.

The Black Knight rocket and its system are then outlined; both its history and the main technical details are discussed. Finally some of the research problems are discussed in more detail with reference to the flight results obtained from Black Knight flights.

Astronautics and Guided Flight Section

SOLID PROPELLANT ROCKET MOTORS

W. R. MAXWELL and G. H. YOUNG

To be given on 6th December 1960

The main types of solid propellants used in rocket motors are first described, with particular emphasis on the double base and composite propellant types developed in the United Kingdom. The various methods of manufacture are very briefly discussed. Recent developments in improved performance and extension of burning rates are described. The limitations imposed on current solid propellants by the restricted range of oxidisers available and by the requirements of safety in manufacture, and use, are illustrated by examples of extruded cordite, pressed charge and plastic propellant compositions in current use. The burning rates, performances, temperature limitations, and particular advantages of each type of propellant are considered briefly.

The special requirements of very large charges in respect of physical properties and manufacturing simplicity have focused attention on case-bonded charges which can be slurry cast, usually with an elastomeric binder based on synthetic rubber; some typical examples are described. It is concluded that future developments in solid propellants will be in the simplification of manufacturing procedures, the extension of case-bonding, and the incorporation of new chemical systems to enhance performance; some possibilities are outlined.

Following on the account of solid propellants consideration is next given to the factors which influence the performance of the rocket motors which use them. It is shown that the main design problem is to associate the maximum amount of propellant with the minimum amount of hardware. The problems of charge design which this involves are discussed and reference is made to problems of ignition and unstable burning. A brief account is also given of the relative importance of specific impulse and propellant density. Problems associated with the development of light-weight hardware, including nozzle materials and thermal insulants are described.

The problems associated with achieving high reliability in combination with high performance are briefly discussed and reference is made to methods of inspection of filled motors and the use of the strand burner for ballistic control.

Finally, some of the recent studies which have been made on the possible uses of solid propellant motors for the first stage boosters for space research vehicles are referred to and some of the problems involved are discussed.

Graduates' and Students' Section

B.O.A.C. AND THE FUTURE

C. ABELL

To be given on 14th December 1960

The lecturer begins by outlining B.O.A.C. organisation and how it works, particularly in relation to firm and long term planning. The next section examines the past results in terms of traffic carried, fleet strength, profit and loss, and extrapolates all of these into future years. There is then a discussion on how very long term plans are gradually firmed up to become immediate firm plans for the next few years. The Lecturer goes on to discuss the effect of developments in the Industry, on the future of the airline and relates, in particular, to the possibilities of Supersonics, the possibilities of all large freight fleets, and the possibilities of very low fare subsonic aeroplanes. Finally reference is made to development of equipment and its effect on the operation of the airline; particularly it refers to automatic pilots and the possibility of blind landing.

FOURTH LANCHESTER MEMORIAL LECTURE

BRITISH UNIVERSITIES AND AERONAUTICAL RESEARCH

PROFESSOR A. D. YOUNG

To be given on 22nd December 1960

The lecture begins with some remarks on the pertinence of the topic chosen for a lecture dedicated to Dr. Lanchester. A brief historical review is then given of the development of University teaching and research activities in Aeronautics from the early years of the century to the present. An imaginary but typical modern University Department of Aeronautical Engineering is described with particular reference to numbers of students, staff, laboratory equipment and budget. Some of the many research topics to which the Universities have made and are making important contributions are then briefly discussed. These are to illustrate the scope and tendencies of University research against the background of current developments in Aeronautics. Among the subjects referred to are such topics as the structure of the turbulent boundary layer and the measurement of skin friction, flow separation phenomena, boundary layer control, boundary layer drag and heat transfer at high speeds, aeroelasticity, the application of matrix methods to structural analysis using digital computers, structural damping, secondary flows in propulsion machinery, the rotating stall and jet noise.

The lecturer concludes with some general remarks on the relationships between the Universities, the Government Research Establishments and the Industry and on the future of University research. The importance of maintaining on the one hand the independence of the Universities and on the other hand the closest contact between them, the Government Establishments and Industry is strongly emphasised.

The Journal of the Royal Aeronautical Society

INCORPORATING THE INSTITUTION OF AERONAUTICAL ENGINEERS AND THE HELICOPTER ASSOCIATION OF GREAT BRITAIN

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Relationship Between Theory and Practice in Aircraft Structural Problems*

by

A. J. TROUGHTON, M.A.(Cantab)., A.F.R.Ae.S.

(Chief Structural and Mechanical Engineer, Sir W. G. Armstrong Whitworth Aircraft Ltd.)

1. Introduction

When designing an aircraft structure, the structural engineer has to achieve given standards of airworthiness in respect of strength, stiffness and fatigue life while ensuring that the final weight is as low as possible to give operating economy or flexibility. At the same time he must ensure that a minimum of design effort is used, manufacture is as easy as possible, the serviceability is good, the aerodynamic shape is retained, aeroelastic effects minimised and so on. The structural engineer of today has to ensure that the fatigue life is satisfactory, together with the older concept of purely static strength, and if he has resorted to the use of fail safe principles he has to establish those characteristics as well.

The designer has three tools at his command to ensure structural airworthiness. The first is the ability and experience of the senior draughtsmen who create and scheme the original structures. The second is the stress teams who carry out theoretical stress checks after establishing the design loads and evaluating possible load paths. The third is the test laboratory where actual practical tests are conducted on detail components or full-sized aircraft.

Of the three, the first is most valuable and hence it is essential to maintain an experienced draughting office. During periods when aircraft structures were fairly well fixed in their concept and design, the know-how of previous designs would carry a design office through to a successful conclusion with a minimum of stressing and testing. Typical examples of this approach are the series of U.S. transport aircraft. However, new structures to meet high speeds and temperatures tend to discount experience and the current policy of rationalising design loads to actual conditions instead of rule of thumb safe loads means that more accuracy is essential.

The second string to the bow is the Stress Department whose theoretical evaluations are carried out by using the latest theory tempered with the firm's own practical test experience. The Stress Department can optimise the structure once the critical loads are evaluated and should control the aircraft's structural design. Their limitations are the difficulties of establishing load paths within the structures—overall design loads are fairly well laid down nowadays—and finally an accurate stressing method which is not too lengthy so that they can be in a position to still influence the design. The use of more advanced techniques such as the computer has eased some of the difficulties but the introduction of fatigue and fail safe design which is not yet amenable to accurate theoretical evaluation has more than offset stressing advances.

The last string is the works test laboratory which has steadily grown over the years from a materials evaluation team to an organisation capable of testing every component up to complete aircraft. The great weakness of a test laboratory team in influencing early design is that it has to await production of specimens—which in themselves are dependent on design finalisation. The essential purpose of a laboratory is to ensure that all its past test knowledge is available for initial stressing and that it speeds component testing to ensure prototype safety without limitations and that any modifications can be introduced as early as possible. The actual development of an aircraft's potential can only be achieved by obtaining its true characteristics in conjunction with final load re-assessment.

This paper sets out to describe the relative merits of stressing and testing in aircraft structural design from all aspects, including the relationship between theory and practice.

GENERAL NOTE

This paper has proved difficult to write because to justify some of the conclusions reached it is necessary to highlight premature structural failures. The examples taken have been supplied by several aircraft firms through their kind permission and are not identified: structural failures are not the prerogative of one nation only. In all

*A Lecture given before the Society on 23rd February 1960.

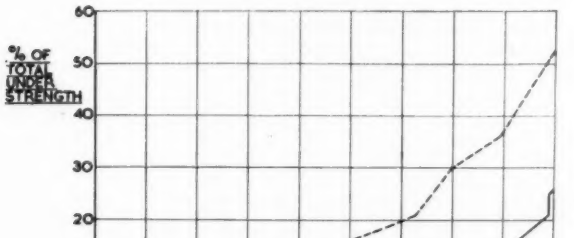


FIGURE 1. Strength test results: below 100 per cent.

cases the failures shown are of initial character and steps were taken to ensure that aircraft in service incorporated modifications.

2. Factors Affecting the Approach to Structural Design

In deciding the best way to prove the structural design of an aircraft, four points must be considered, namely:—

(*i*) the accuracy obtained by the results
(*ii*) the cost
(*iii*) the ability to develop the aircraft
(*iv*) the time factor.

These will be dealt with one at a time.

2.1 ACCURACY

As weight is so important (on the Argosy one pound of weight is worth £50 a year in revenue) it is essential that the method employed is as accurate as possible so that the final structure can just meet the required conditions without excess weight. Also, to satisfy both military and civil airworthiness authorities it is essential that the method employed gives a firm guarantee of the answer. All the evidence to date is that stressing alone will not give the desired result. Fig. 1 shows the results obtained from 100 detail static strength tests and 67 full scale static strength tests using published R.A.E. information and unpublished industrial results—initial failures only.

It will be seen that 20 per cent of the full scale tests failed at up to 70 per cent of the design loads and 45 per cent failed at up to 95 per cent of the design loads. The detail tests show somewhat better values but even in these 18 per cent fail below 95 per cent of the design loads. The reverse side of the coin—those greater than 100 per cent—are shown in Fig. 2 for the same series of tests. This is not so clear cut a case as failures below 100 per cent as the current fatigue emphasis tends to give static strengths in the 120-130 per cent region. However, taking 130 per cent ultimate strength as the maximum desirable it will be seen that 44 per cent of all detail tests were above that figure. Only 8 per cent of full scale tests failed above 130 per cent of the design ultimate. The two curves

FIGURE 2. Strength test results: above 100 per cent.

highlight two points: the wide variability obtained on initial detail tests and the fact that even with that test knowledge incorporated the stressing method used cannot even yet be relied upon to give a reliable guide to static strength.

In the fatigue field the current position is just as difficult when predicting lives from theoretical considerations alone. Fig. 3 shows typical fatigue results for different joints plotted on an *S-n* curve together with a standard acceptability curve: Heywood's light alloy lugs curve with a mean stress of 13,000 lb./in.^{2}. It will be seen that a series of tests on a joint can have very wide scatter indeed as regards life depending on detail design. The maximum scatter on one series is over 7: 1 and the ratio of the mean lives is 15: 1 for one joint and 3: 1 on the other. The problem is complicated by the fact that the designer is resorting to "gimmicks" to increase fatigue life such as the use of M_0S_2 anti-fretting grease,

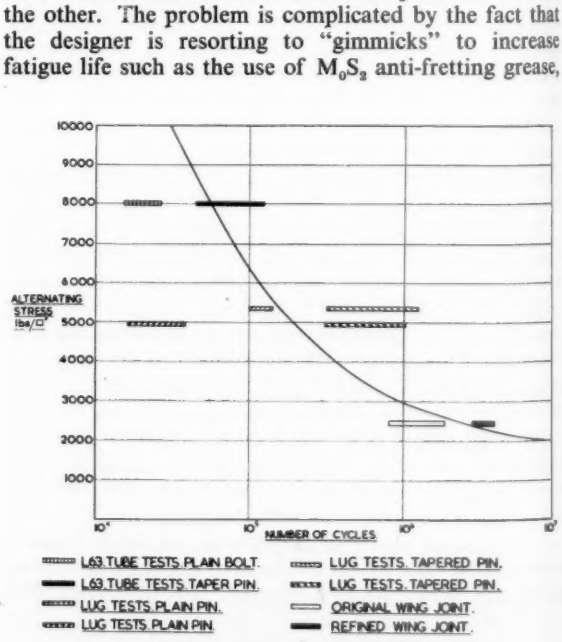


FIGURE 3. Typical fatigue results.

anti-fretting shims, all of whose beneficial effects are not calculable for a given type of joint. There is a strong school which recommends that these "gimmicks" are not put in the test specimens so that the results are guaranteeable and the aircraft will be somewhat better, but unknown. I am not a firm supporter of this approach as I consider that we must use any advantage we can to extend fatigue lives. Thus when the safe life approach is used testing is essential and as will be demonstrated later the advent of fail safe has emphasised rather than relieved the testing commitments.

2.2. COST

In many cases the designer uses the stress team to do the initial design and then tests all the components resulting in an expensive design bill; stringent military requirements have led to this approach. It is essential that the designer can weigh the relative costs of testing and stressing so as to come to as economical a solution as possible. It is useful to maintain man power and cost statistics as a background to such decisions.

Figure 4 shows the man power expended on the Argosy from the initial commencement of draughting until the C. of A. which is due soon. It can be seen that the total effort expended by the design staff will approach 2·5 million man hours, of which the draughtsmen contributed 1·28 million man hours and the test department 0·375 million man hours. It will be noted that the stressmen/draughtsmen ratio is rather low so that the test laboratory in this case has had to play a large role. Even not including capital costs of equipment, specimen costs, industrial test laboratory labour charges and so on, the cost of structural approval has involved many hundreds of thousands of pounds. A small sum compared with some aircraft but a large charge on each aircraft produced. It is considered essential that designers should watch the financial implications of testing and stressing. There is ample scope for technical management in this field so that resources are used efficiently. The designers optimise structures but rarely optimise their use of their design facilities. Modern facilities themselves are expensive. The A.W.A. complete static/

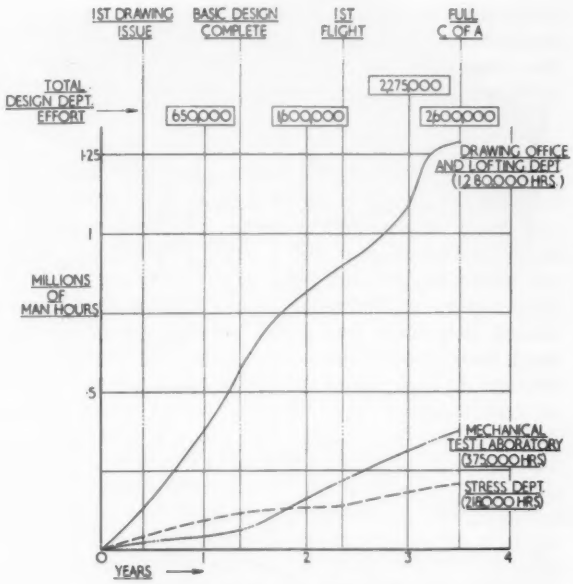


FIGURE 4. Argosy man power.

fatigue water tank facility is shown in model form in Fig. 5 and this, coupled with a full scale fatigue laboratory, cost over £200,000. The next generation of aircraft will need even more expensive facilities when thermal effects are paramount. This paper will attempt to give methods of obtaining the maximum value for money. One facet of this is the importance of a system of recording accurately test information and its relationship to theory for future reference both of one's own tests and the myriad of reports from all over the world. A good co-ordinator, abstractor and data sheet compiler is worth his or her weight in gold in money saved in the long run. Another essential is to retain experienced staff so that the test knowledge permeates the design team. In the test laboratory money can be saved by designing flexible multi-purpose test facilities, using one's own rigs rather than expensive proprietary fixed bed machines and

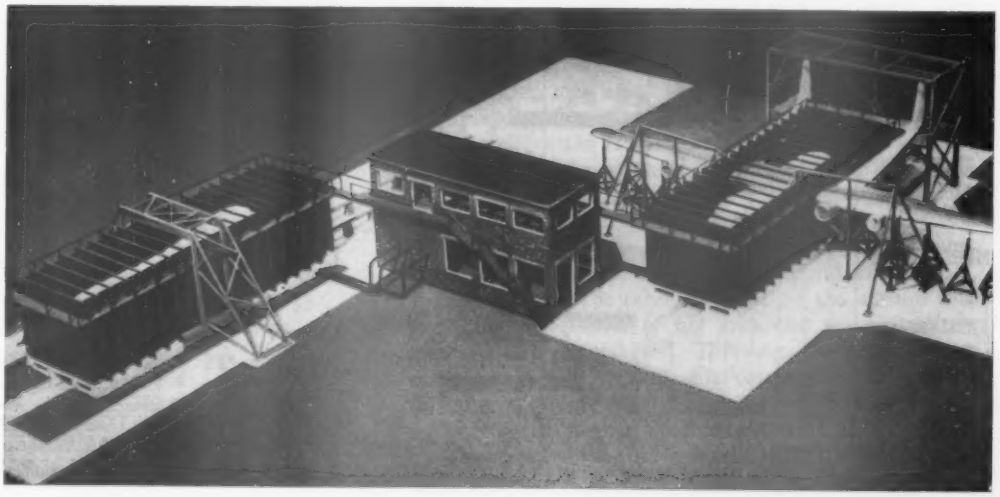


FIGURE 5. Model of A.W.A. tank facility.

developing new techniques. While strain gauging is invaluable a reliable alternative which would reduce the man power cost and trouble is needed for large tests with thousands of measuring points.

2.3. DEVELOPMENT

It is an unfortunate fact that aircraft weights rapidly increase during design due to added requirements, extra equipment, more realistic design and so on. So it is important to develop the aircraft structure to its full potential. The stressman can do this in many ways, *e.g.* using a more refined method than his initial assumptions, finding alternative load paths, solving redundancies or using more sophisticated calculations for his allowable stresses. In many cases the design loads can be reduced as knowledge of the aircraft's actual characteristics grows. However, actual tests are the best guide to developing an aircraft's potential as by continual modification after a series of tests it is usually possible to reach a considerable improvement in strength for a small weight increase. The cardinal rule is to modify after failure and press on; it is no good stopping at the first failure as the aircraft's role is bound to extend. Two other important factors in developing an aircraft are its limit of serviceability and the determination of suitable maintenance schedules and the modern structural team regards these as important as the more classical features such as static strength. One essential aid to successful development is structural integrity flight testing to provide accurate loads.

2.4. TIME

The time factor is paramount in aircraft design as aircraft rapidly become obsolete. All time aspects must be watched. Too detailed an initial stressing can hold up aircraft design by putting back the first issue of drawings. Too much reliance on a test programme may mean the information is too late to allow a modification programme to be carried out and finally, unless a stronghold is put on production to allow the first prototypes for testing the results may be too late and final certificate of airworthiness held up until some period after the actual flying is completed. Fig. 6 shows the relationship of the Argosy structural testing and the main stages towards certification. The continuing detail test programme reflecting minor improvement throughout the design should be noted. One significant test specimen was the short fuselage length which was tested in the early design stages and proved valuable in design. The static tests have to be completed before the granting of C. of A. and the fatigue tests have been completed to give at least 5,000 hours' safe life by that date.

All aspects of testing and stressing from these four standpoints will be discussed.

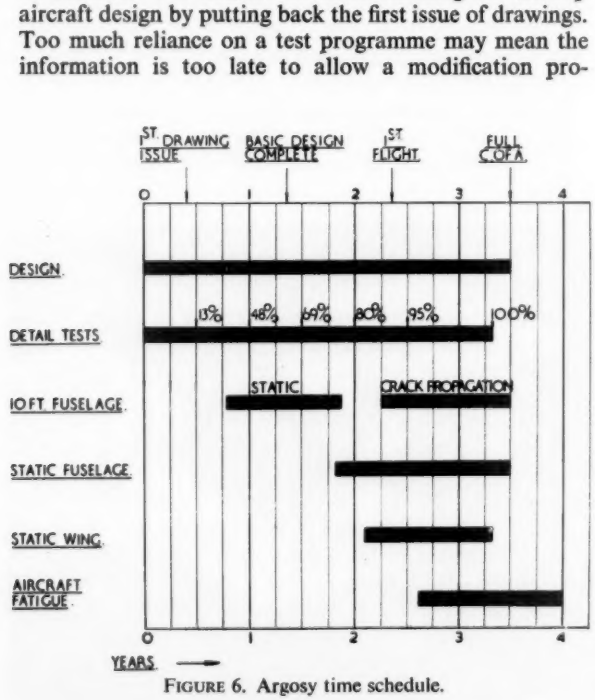


FIGURE 6. Argosy time schedule.

3. Research Testing

It is essential to carry out research testing continually so that if a new design appears the actual design work is not held up by lack of technical data. There are six main fields of research testing which a design department should initiate and foster, namely, materials information, optimum structures data, detail stressing problem evaluation, routine component development, engineering research and inspectional technique research.

The materials problem is self-evident, arising from the lack of full information from the supplier particularly if temperature effects are significant. The N.A.T.O. handbook is very valuable but the progressive manufacturer must keep ahead on new materials. In the fatigue field there is little valuable information with representative specimens to cover cumulative damage, crack propagation and so on. In non-metallic fields there is a dearth of information on the structural properties of plastics and the materials research laboratory must keep

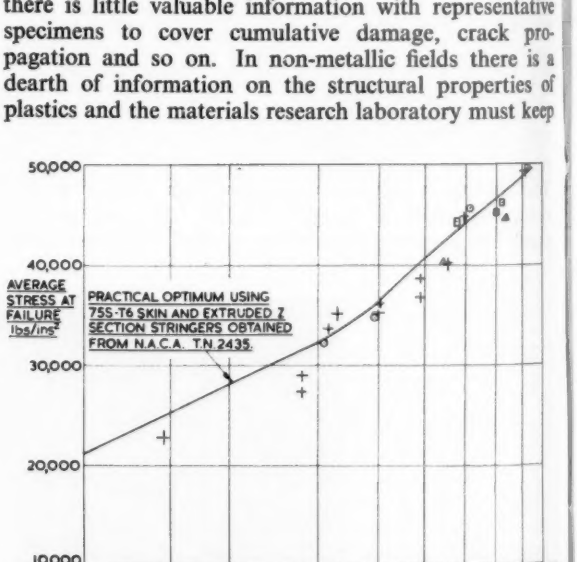


FIGURE 7. Optimum compression panels.

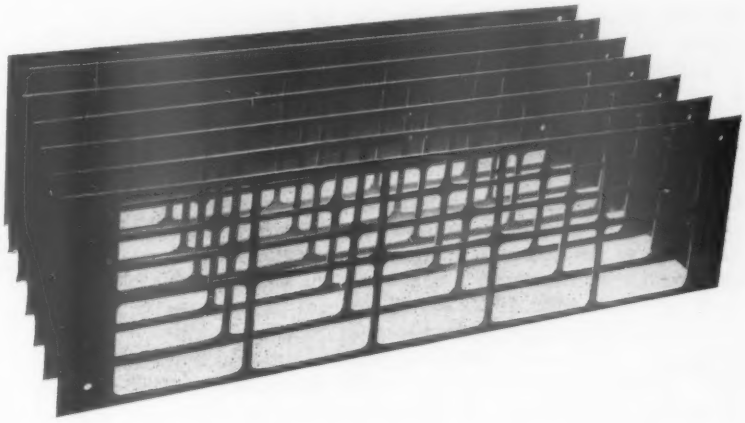


FIGURE 8. Routed sheet construction.

a strict watch on pretreatments, paints and finishes and suitable anti-fretting compounds.

The optimum structures problem is to provide test data to back up the growing theoretical work on this subject. A system should be introduced so that *all* static test results are fed back into optimum structure data so that the design can be evaluated. It is true that in actual aircraft design theoretical optimum structures are never welcome—too close a stringer pitch and so on—but the Industry must work towards optimum. Fig. 7 shows typical compression panel results compared with optimum information. It will be seen how close one can reach towards the goal and with this type of information project and detail stressing can be relied upon.

The evaluation of detail stressing problems is largely covered by the excellent series of Royal Aeronautical Society data sheets. However, particularly in practical details there are many problems which must be actively pursued by firms, *i.e.* buckling of flanged hole webs, strength of club feet in tension and local crippling. Possibly the introduction of aircraft groups may lead to rationalisation of detail data sheets.

The routine component evaluation is again self-evident but valuable. It is probably in the field of fasteners that this is most important: manufacturers must press so that new ideas are rapidly accepted by the authorities or we shall be at a disadvantage in comparison with foreign manufacturers.

Engineering research is valuable as many current production techniques are initially linked with the final structural capabilities. Examples of this are metal-to-metal bonding, chemical milling, steel aircraft constructions and surface finishing.

Research into inspectional techniques must be carried out as a by-product of structural testing. The methods learnt and the critical locations are essential for the drawing up of maintenance manuals and to assure a customer of the integrity of your aircraft.

Most of the testing in the above categories are routine and do not need illustrating. One example of research testing is given in Figs. 8 and 9. Fig. 8 shows a spar flange specimen which was made of bonded laminates, the internal ones sculptured for stabilisation with minimum weight with solid face sheets. This design was tested at varying temperatures in compression with varying adhesives. The results are shown in Fig. 9. These are more valuable than the conventional shear or peel tests as the buckling effects are accurately reproduced.

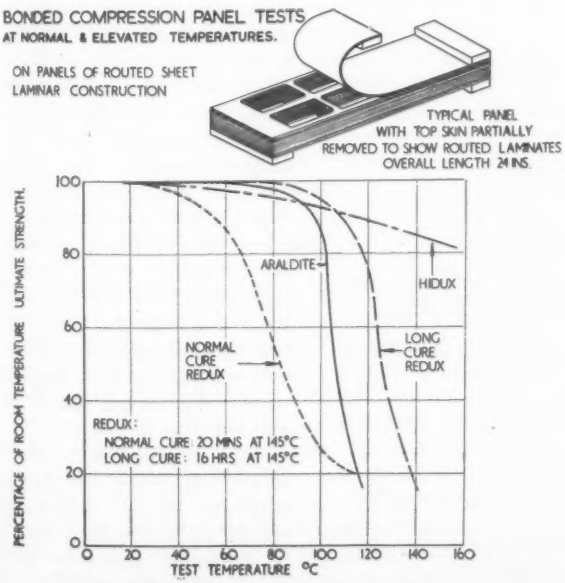


FIGURE 9. Adhesives at elevated temperature.

4. Stressing Technique

There are three main stages in stressing:—case load evaluation, load path and actual stress evaluation and finally allowable stress evaluation. The principal aim of a stressman should be to influence the structural design so he can actually stress it with confidence. In practice this means providing ascertainable load paths with already tested structural features.

Load path simplification is important—fail safe consideration may affect this slightly but Ford's slogan "Simplicate and add Lightness" still stands. Coupled with simplification is the elimination of offsets and the introduction of symmetry wherever possible; the drunken behaviour of a channel in bend compared with an I-beam cannot be too highly stressed. The load paths particularly on details should follow shear attachments rather than tension paths, especially round corners.

The stressman is rarely in error on the tension side providing the pitfalls of net area and notch sensitive materials are remembered. However, the negative case on the primarily tension side can sometimes provide difficulty, plate tension longerons which may buckle under negative normal acceleration are a case in point.

The compression side is usually closely stressed and is

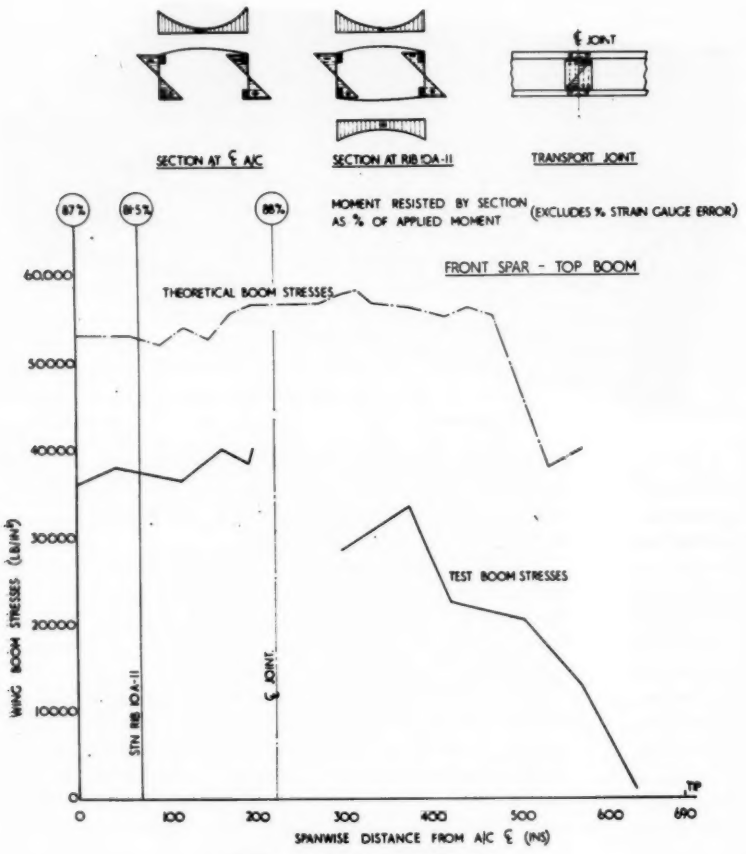


FIGURE 10. 2-Spar wing stresses.

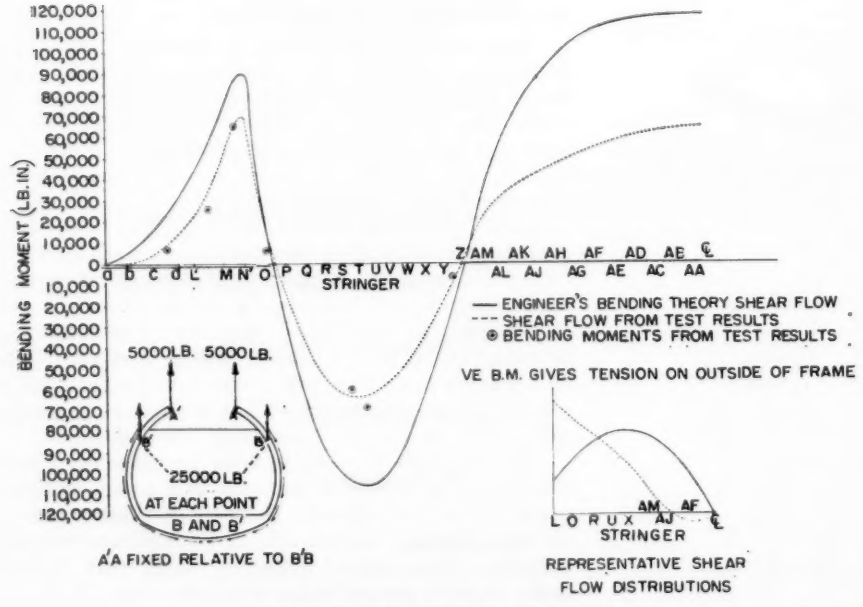

BENDING MOMENT (LB.IN.)
STRINGER
— ENGINEER'S BENDING THEORY SHEAR FLOW
--- SHEAR FLOW FROM TEST RESULTS
⊕ BENDING MOMENTS FROM TEST RESULTS
VE B.M. GIVES TENSION ON OUTSIDE OF FRAME
5000LB. 5000LB.
25000 LB. AT EACH POINT B AND B'
A'A FIXED RELATIVE TO B'B
REPRESENTATIVE SHEAR FLOW DISTRIBUTIONS

FIGURE 11. Wing pick-up frame design.

covered by detail tests. Here, as will be shown later, the weakness is usually the supporting structure. It is sometimes better to aim for designs near to the U.S. test-proved optimum designs for compression flanges as the full theoretical treatment becomes lengthy.

One problem is die-away loads. Wherever possible, the stressman should convince the designer they are unacceptable from a weight and stiffness standpoint. If essential they can be stressed by the picture frame method or equivalent stringer die-away method for a first approximation. The best way is to programme a generalised computer programme for cut-outs.

There is no doubt that the normal engineers' theory of bend and shear is fully acceptable and gives conservative results for conventional structures, providing allowance is made for special features such as root constraint, sweepback and die-away. A stress office builds up arbitrary figures based on tests to cover these problems. Fig. 10 shows a typical two-spar wing with the engineers' theory stresses and those measured. In addition spot points are shown for the ratio of the measured bending moment from strain gauges/actual applied bending moment when all the structure is included. This gives 87 per cent and 82½ per cent which falls into the usual bracket of up to 15 per cent missing. The conservatism of simplified stressing is self-evident.

One particular feature to watch is strain compatibility. Many full scale tests fail in the supporting structure through lack of appreciation of this basic fact. One other simple rule is to draw the complete bending moment and stress fatigue diagram. One undercarriage tee-shaped fitting failed in the tee-junction at 42 per cent after being in service some months, due to lack of appreciation of the full shear force diagram; the centre section webs of wings or tailplanes between fuselage pick-up points under asymmetric loads are another case in point.

A stressman should always make bracketting assumptions when in doubt. Rib design should allow for the torque to be taken off partly in batho shear and partly in differential

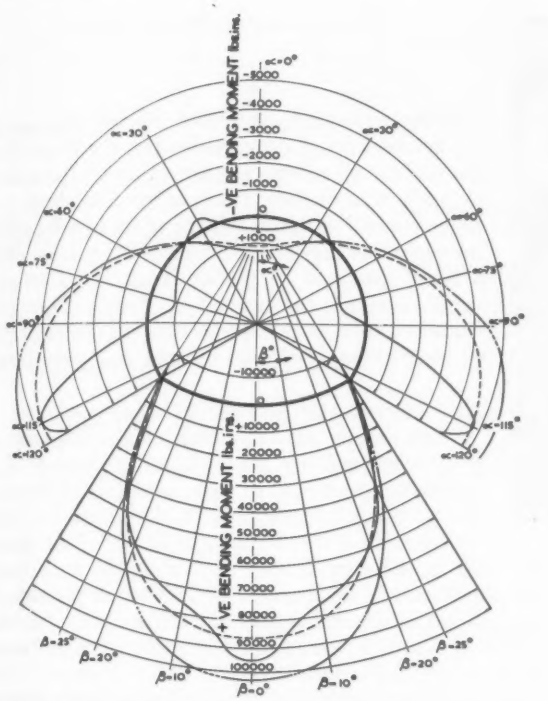

-VE BENDING MOMENT lbs.ins.
+VE BENDING MOMENT lbs.ins.
NOTE: POSITIVE B.M. PUTS TENSION IN OUTER FLANGE.
---- SHOWS THEORETICAL BENDING MOMENT (INCORPORATING 85% RELAXATION FACTOR.)
—— SHOWS THEORETICAL BENDING MOMENT.
—— SHOWS ACTUAL BENDING MOMENT.

FIGURE 12. Frame under pressurisation loads.

bend with varying proportions. A good case of using bracketting assumptions is in frame design. Fig. 11 shows theoretical and measured bending moments for a wing pick-up frame under the loads shown. The two reactant shear distributions—engineers' theory and measured—are shown together with the frame bending moments calculated from strain energy for the two distributions and the measured bend stresses. The agreement is very good for the measured non-engineers' theory distribution and thus it pays to assume different shear distributions initially when frame stressing. Rapid initial frame stressing is very difficult and most firms have their arbitrary rules such as divide by 2 or multiply by 0·75. Fig. 12 shows measured and estimated frame bending moments for a non-circular fuselage under pressure only. The test justifies the arbitrary scaling figure of 85 per cent used in the design. (This figure was used after calculating that the frame would carry 50 per cent of the pressurisation loads, the rest being in hoop tension in the skin.) The correctness of the frame bending moment variation can also be seen.

Stressing for fatigue has three basic rules:—(*i*) evaluate fatigue stress levels spectra for mean operational conditions—never work to conventional ultimate strength reserve factors, (*ii*) select design stresses based on the best test data available and add one's personal factor, and (*iii*) have rigid design office rules to obviate the detail errors which prevent one reaching the typical life normally obtained at that stress level.

Stressing for fail safe structures is a widely differing procedure. The approach can vary from the simple acceptance of the airworthiness authorities standard of strength required with a damaged structure, to a sophisticated statistical approach balancing crack propagation against loads arising during a given inspection period.

One example of wasted effort in a stress office is having to repeat calculations frequently as the weights and aerodynamics change. It is best to be firm and assume that the aerodynamic department will change its loads until the day before certification and thus keep down frequent re-assessments. Since it is possible to increase the strength of aircraft structures substantially by testing with little weight penalty the question automatically arises as to whether it is possible to design an aircraft for a lower weight than that at which you wish to certify it. This method is attractive in giving a lower percentage structure weight and therefore there seems considerable justification for it providing one is firm in testing throughout. The policy is difficult to maintain especially with sub-contractors who have contractual obligations to meet and who are extremely worried about reserve factors lower than one appearing on their documents before testing. The advantages of this policy are so great and it has been proved so often that I feel that it is best to maintain the project weight right up until the final check stressing and testing.

5. Correlation Between Theory, Structural Test and Flight Test

Figure 13 shows fighter wing stresses at 6*g* using the original design engineers' theory of bend and measured results with strain gauges from a full scale wing test and actual flight results during symmetric pull-outs. The gauges are external only, so local bend stresses on

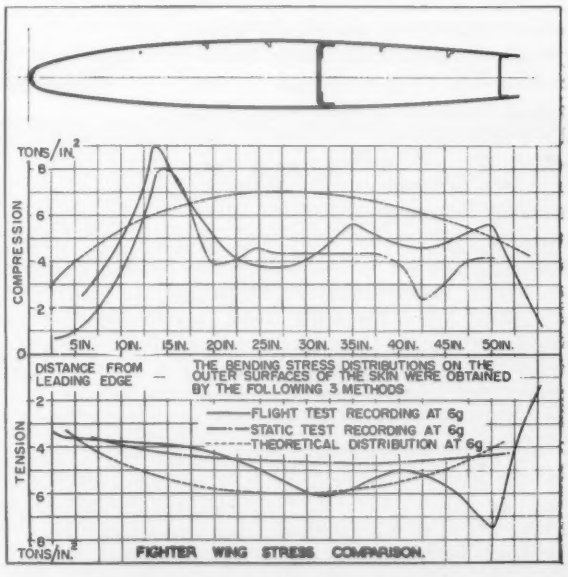

TONS/IN.²
COMPRESSION
TENSION
5IN. 10IN. 15IN. 20IN. 25IN. 30IN. 35IN. 40IN. 45IN. 50IN.
DISTANCE FROM LEADING EDGE
THE BENDING STRESS DISTRIBUTIONS ON THE OUTER SURFACES OF THE SKIN WERE OBTAINED BY THE FOLLOWING 3 METHODS
FLIGHT TEST RECORDING AT 6g
STATIC TEST RECORDING AT 6g
THEORETICAL DISTRIBUTION AT 6g
FIGHTER WING STRESS COMPARISON.

FIGURE 13. Fighter wing stress comparison.

FIGURE 14. Spar boom static failure.

the compression side are magnified. However, the shape and character of the stress pattern is similar in all cases.

6. Design Lessons Learnt as a Result of Testing

The following six lessons are usually highlighted during testing (in addition there are several fatigue lessons learnt as described later):—

(*i*) Simple design failings.
(*ii*) Limit of serviceability.
(*iii*) Unexpected stress distributions.
(*iv*) Unexpected load paths.
(*v*) Poor compression supporting structure.
(*vi*) Design assumptions too severe.

6.1. SIMPLE DESIGN FAILINGS

These usually occur on account of the sheer volume of work associated with aircraft design and should be the special province of the design section leader. Fig. 14 shows a typical case. Here a main spar boom—some years ago now—failed at 92 per cent of the design load due to inadequate allowance for a bolt hole added after initial stressing was completed. This is the net area problem but there are many others, namely, braced framework lines of action may not intersect due to assembly problems, a common failing. In the fatigue field there are many quite small features which may lead to premature failure such as inadequate radiusing of holes or sharp edges, rapid changes of cross-section, rough finish, eccentricities and high die-away loads. It is the sensitivity of fatigue life to these features and the tremendous number of components to check in one aircraft that leads to fail safe design and/or full scale testing.

6.2. LIMIT OF SERVICEABILITY

In practice this is more important than the theoretical proof or ultimate strength. The stressman who automatically checks the proof strength on a purely 0·1 per cent proof stress basis must not forget that the real design criteria is also set up by the ability to move control rods, the occurrence of fouls between control surfaces and the main structure and the general ability of the aircraft to get home after a proof load and be quickly back in full operational condition the next day. Testing alone can demonstrate these features as they depend critically on the picking up of load by subsidiary structures.

6.3. UNEXPECTED STRESS DISTRIBUTIONS

This mainly occurs through the maintenance of strain compatibility throughout the aircraft and this section is mainly to cover unexpected induced effects which the stressman had not allowed for. An example of this is shown in Fig. 15. This shows the effect on adjacent frames of loading the main wing pick-up frames of an aircraft. In the case demonstrated the aircraft is loaded at the end frames only but the unloaded frames in the region of the wing pick-up frame have the stress distribution shown at the bottom. This carry-over factor still exists when all frames are loaded and provides an added design load.

6.4. UNEXPECTED LOAD PATHS

These arise from redundancies in the main structure which are not included in the original theory or subsidiary structure taking unexpected load due to strain compatibility. Fig. 16 shows a typical case. Here the theoretical loads are compared with the measured strain gauge values for an engine mounting. It will be seen that

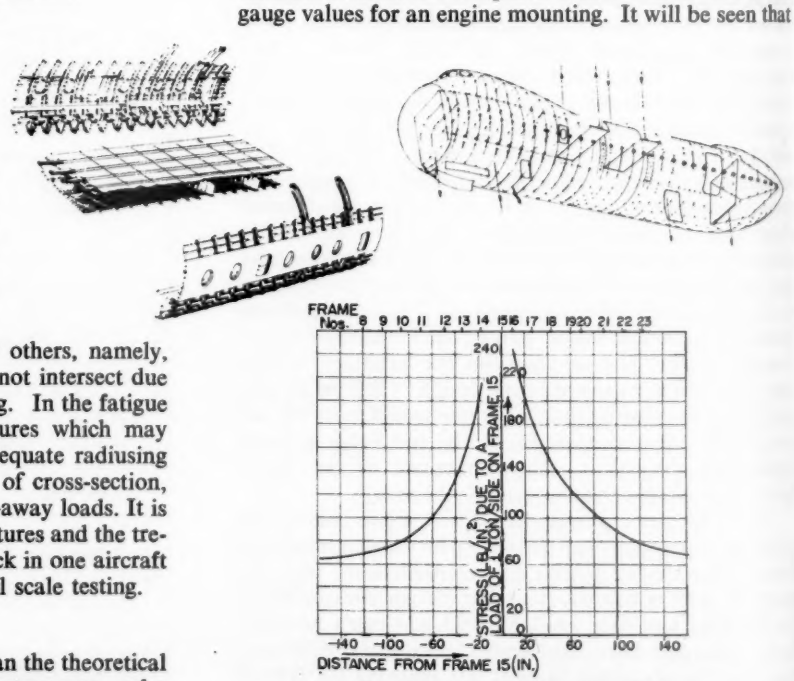
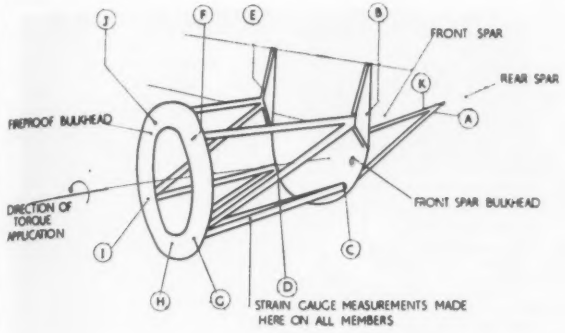


FIGURE 15. Frame carry-over effects.



(1) HIGH INERTIA CASE

MEMBER	THEORETICAL LOAD	TEST RESULTS DIRECT LOAD LBS.	TEST RESULTS BENDING MOMENTS LBS-INS
JE	16,500	9800	*
FB	10,000	13,300	450
GB	20,700	17200	0
GC	-30,100	-25,400	400
IE	23,100	18,000	*
ID	-34,800	-25,600	700
HC	-840	-420	*
HD	+840	-520	210
CA	-32200	-22,000	6200
DK	-34,000	-26,000	3100

(2) HIGH TORQUE CASE

MEMBER	THEORETICAL LOAD	TEST RESULTS DIRECT LOAD LBS	TEST RESULTS BENDING MOMENTS LBS INS
JE	12,700	3000	*
FB	-7,000	2700	8,104
GB	-30,000	-7,800	4500
GC	33,100	7600	8,700
IE	41,700	14,500	*
ID	-49800	-19,300	9,400
HC	22200	0	1400
HD	-22,100	0	*
CA	57,200	10600	1070
DK	-70,500	-20,400	2500

* NO PROVISION WAS MADE FOR THE MEASUREMENT OF BEND STRESS
– VE SIGN DENOTES COMPRESSION.

Comparison between load distribution obtained on test and tube loads calculated assuming pin jointed theory.

FIGURE 16. Engine mounting: member loads.

the agreement between theory and practice is fairly good for vertical inertia cases but very far out for high engine torques.

It will be seen that the theoretical values are usually conservative but in the high engine torque case the loads in major rear struts were substantially lower than expected. The greater accuracy in the vertical sense probably arises from the fact that the pins are horizontal and thus local bending moments are reduced. However, this highlights one point and that is that detail static or fatigue tests are likely to be quite misleading if based on the theoretical loads in this case. Complete engine mounting testing is essential. Unit load tests for torque, thrust and inertia are desirable to cover engine development.

Another interesting case was the pyramidical structure for supporting an undercarriage which strides the wing torsion box cut-out for the undercarriage wheels. The front struts of the pyramid are designed to take the undercarriage drag stay loads being in line with it but the back two struts have theoretically zero load, being just stabilisers. However, the aircraft had pin bearing failures in the rear tubes and the full scale fatigue specimen showed incipient failure. Flight testing and complete aircraft fatigue testing showed a considerable load in these struts. This was found to be due to the pyramid taking the wing torque, *i.e.* acting as the missing bottom skin of the box and also picking up wing bend deflections.

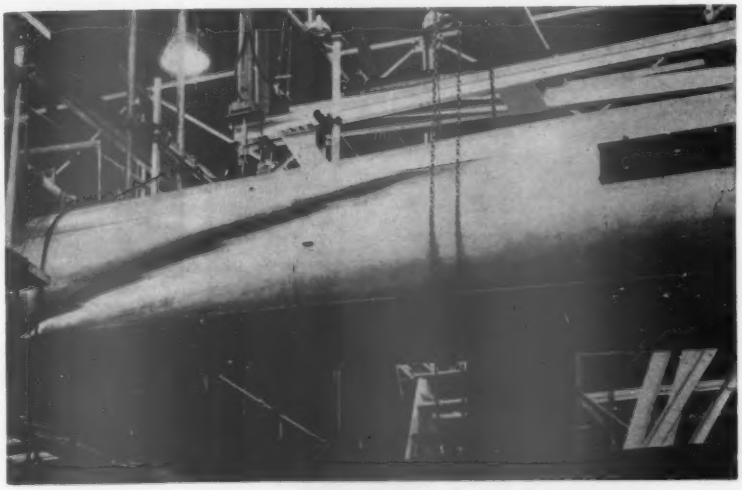


FIGURE 17. Fuselage shear buckle.

6.5. POOR SUPPORTING STRUCTURE

While the theory of compression flanges is better understood these days the theory behind the supporting structure requirements is not so clear cut. Fig. 17 shows a fuselage permanent buckle which was caused by the failure of the stringer/frame combination to restrict shear buckling to the individual bays concerned. The panel aspect ratio may have contributed to the failure as did certainly the fact that the frames were only connected to the stringers through the stringer flanges. Local additional stiffeners increase the buckling stress to above ultimate with a small weight penalty. Fig. 18 shows permanent buckling of a wing skin under torsion loads. Here the deep torsional skin buckles have twisted the restraining rib flanges so that the buckling spreads from one panel to the next, giving a general failure. No theoretical treatment is available for either of the last two types of failure and the only safeguard is for the designer to provide a reasonable relationship between his skin gauges and the strength of his supporting structure.

6.6. DESIGN ASSUMPTIONS TOO SEVERE

The stressman, by nature, is usually conservative in his assumptions and thus it is valuable to use detail tests to give more accurate initial assumptions. Fig. 19 shows a typical case. Here detail tests were carried out using an actual wheel on a floor panel to obtain the tyre spread with increasing load and also the effective number of floor stiffeners taking load. By this means the actual stresses were computed from the deflections and a series of arbitrary assumptions set up for floor design with vehicular loads which gave considerable weight saving. This is the type of detail test which pays dividends.

In the fatigue field the lessons to be learnt from detail tests are legion and the only way is to set up rules for the drawing office based on experience. Everything from

FIGURE 18. Wing shear buckle.

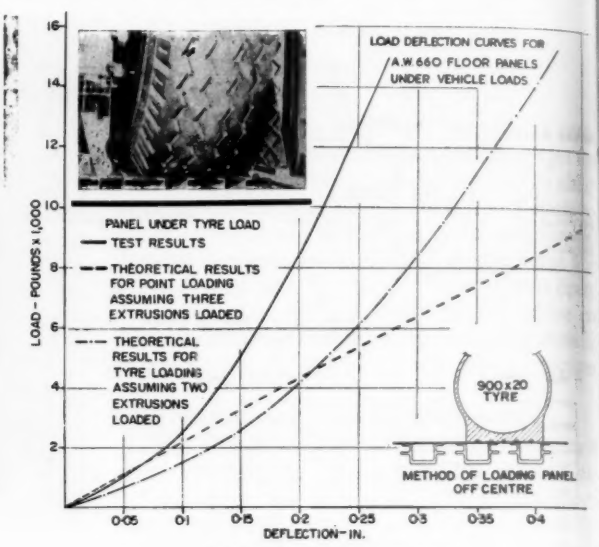


FIGURE 19. Floor panel test.

minor design points to the final method of production affects fatigue. Fig. 20 shows tube socket fatigue results plotted on an *S-n* curve with Heywood's standard curve. (L.A. tubes with varying sockets.) It shows that five $\frac{5}{16}$ in. taper pins in a steel socket are better than five $\frac{3}{8}$ in. bolts in a steel socket. It then shows that eight $\frac{5}{16}$ in. taper pins in a steel socket are better than five $\frac{5}{16}$ in. taper pins in a steel socket. But it also shows that the best is five $\frac{5}{16}$ in. taper pins in a L.A. socket. This is because the L.A. socket has a more comparative stiffness to the tube and thus the loads are shared more equally per fastener. Incidentally it has also been demonstrated that bad reaming lowers the life to about a tenth of that shown on the curve.

Also it can be the unexpected which fails in fatigue. Fig. 21 shows buckling of a concave bulkhead in a pressure container. This buckling was of a snap nature and led to fatigue cracks associated with buckling which is more unusual than the conventional tension failure.

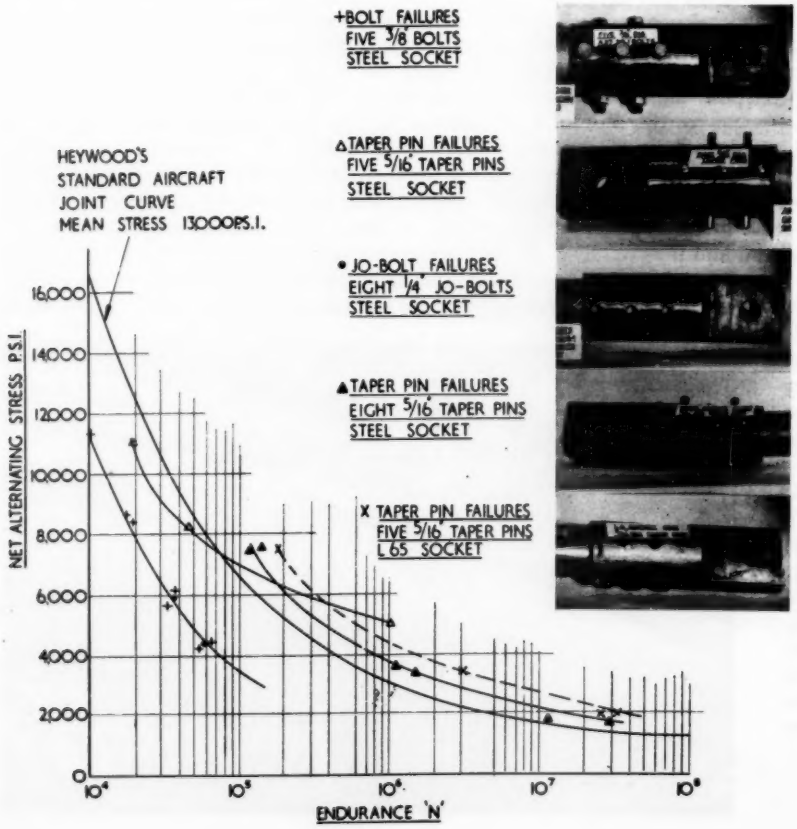


FIGURE 20. Socket fatigue design.

7. The Relationship Between Detail and Full Scale Tests

The detail test will always be essential as it is the only quick way of assessing design within a period which will allow the design to be influenced. There are many detail specimens which are valuable in giving an accurate result such as floor panels, freight lashing attachments, control run details, undercarriages, radomes and small control surfaces, but the majority of detail specimens which involve isolating a structural element, such as a joint or compression panel, must be tested with care. Basically the problem is due to the applied load assumptions being in error and the support conditions being unrepresentative. A classic example of the latter was the case when the detail compression panel failed in the opposite

FIGURE 21. Pressure container bulkhead buckling.

sense to that on the full scale aircraft due to the absence of brazier loads on the detail specimen. This led to higher compressive stresses on the aircraft panels skin side with premature failure due to inter-rivet buckling, a condition never highlighted on the detail tests. However, I am convinced that providing one remembers the limitations, the detail test is invaluable, particularly for comparative work. Fig. 22 shows a typical series of detail tests used for comparative purposes, in this case a main bolted joint. The joint has tapered bushes and the curves show the increase in fatigue life plotted against the degree of pretensioning.

While the detail tests are invaluable for comparative purposes such as scatter, production technique effects and minor material changes, the application of these to design is one of the designer's skills. Evidence at Armstrong Whitworth Aircraft Ltd., suggests a factor of at least 2·0 should be put on fatigue detail tests and possibly 1·25 on static detail tests to reduce them to full scale results with their genuine offsets, local bends, irregular load distributions and so on. However, in the majority of cases the comparative qualitative result has been correct.

A good example of a useful detail test specimen is shown in Fig. 23. This was a parallel section of Argosy fuselage made to check the stresses in a non-circular section. It was completed quickly and proved very valuable in training the production personnel on the type of construction which would follow. It has been used to check frame bending moments, skin stress levels with and without cut-outs and also fail safe characteristics. Speed in construction and testing is essential in this type of detail test specimen.

8. Fail Safe Design

At a first glance it hardly seems necessary to establish the conventional static and fatigue characteristics of a fail safe aeroplane as by its very nature safety is assured. This is misleading because an aircraft, which although strong enough, continued to crack and give trouble would be an embarrassment to airline operators. It is the lost

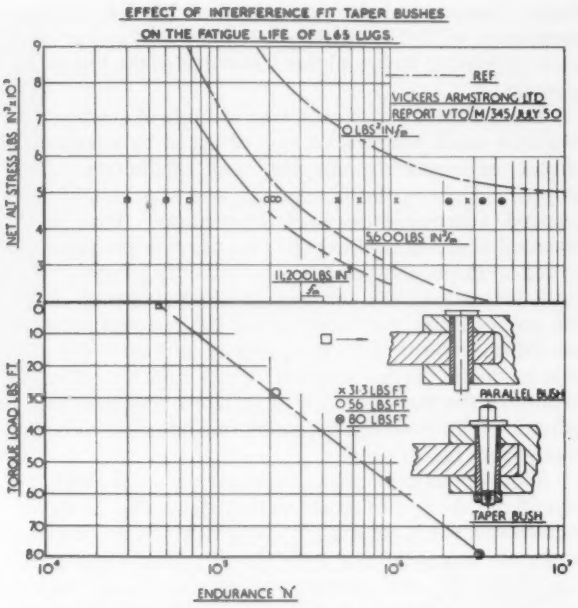


FIGURE 22. Improvement in joint fatigue life by pre-tensioning.

revenue due to aircraft being out of service which is more important than the cost of the actual repairs. Thus I feel a fail safe aeroplane must have a sound safe life and thus the normal type of fatigue testing is still applicable. In the static field even a fail safe aeroplane must still withstand the design ultimate loads so this testing is still valid. In addition fail safe characteristics must be demonstrated. Fail safe can be obtained by a multiplicity of elements or reducing the crack propagation rate. The multiplicity of elements approach is misleading when only a few elements are present in that, unless the remaining load path is purely of a standby nature and thus heavy, the remaining elements will have had an equal

FIGURE 23. Argosy fuselage detail specimen.

fatigue history and thus have limited life left as they themselves will be in the fatigue scatter band and what is more important have a higher stress due to the loss of an element.

The best method of achieving fail safe characteristics is a slow crack propagation rate obtained by a judicious choice of materials coupled with deliberate crack stoppers. Only by this method can one with certainty fly cracked aeroplanes between inspections. Thus it is essential firstly to establish the rate of crack propagation to set up inspection periods and, secondly to obtain the static strength of the unit with cracked elements. Both the rate of crack propagation and the residual strength are difficult to calculate theoretically and therefore it is evident that in the fail safe field there is need for extensive testing in the future before there is an adequate background of information to enable designers to proceed safely from theory alone.

Some practical crack propagation tests will be considered, firstly, full-scale aircraft tests. Fig. 24 shows web cracks which occurred in a fighter where the fuselage longeron necessitated a web cut-out. The right hand side shows the crack pattern on full scale testing with a programme loading which gave a safe life of about 1,700 hours (referred to an arbitrary severe flight plan). The left hand side shows the detail crack stoppers used—a polished hole and a steel reinforcement plate to reduce the stresses in the cracked region—a weight penalty of 3½ lb. The life was increased fivefold by this single modification as shown in the bottom graph. The reason for the unusual direction of the crack propagation is the fact that the cracked area is a region of high taper shear.

Another example of crack propagation investigation is the work carried out in conjunction with the Argosy design. Fig. 25 shows a typical section of the Argosy fuselage. At each frame there is a peripheral crack stopper in the form of a reinforcing strip under the skin. A series of tests has been carried out on a fuselage section to check the efficacy of these bands. Fig. 26 shows typical arrangements of crack detector wires used in the under-water investigations. Fig. 27 shows the

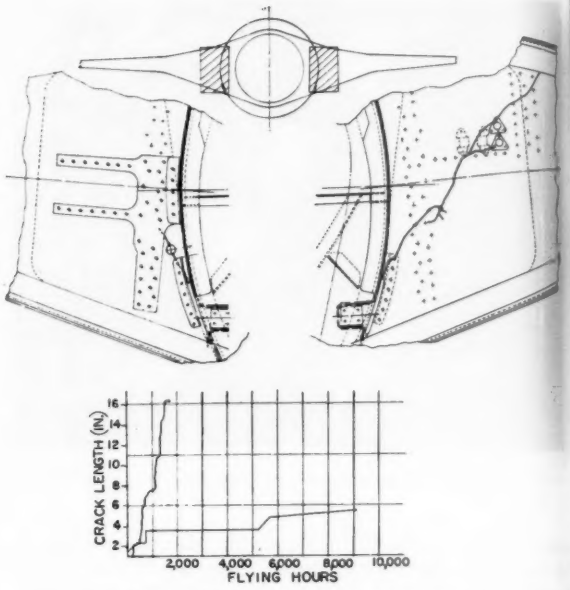


FIGURE 24. Fighter web crack stoppers.

results achieved to date. The main result is that even after initiating many cracks, some up to 16 in. long, and cycling for over 30,000 reversals (*i.e.* twice the full *life* with that crack) no crack has crossed a frame peripheral stopper. The figure shows the relative crack propagation rates of D.T.D. 710 and D.T.D. 746 and also crack lengths which are below critical. One interesting feature is that cracks were introduced after 30,000 reversals to compare with initial cracks to see if a fatigue history has been built up but the crack propagation remained constant. This type of test is invaluable in proving the efficacy of sophisticated crack stoppers.

As part of a basic research programme A.W.A. have been investigating fail safe web design. Fig. 28 shows the test rig. Two spar booms with their associated webs are tested simultaneously and there are four areas of test webs in each wing. The rig is operated by a single jack connected to the Losenhausen equipment and it is used for testing four panels at once, each panel being of the same design. Fig. 29 shows the results for different web constructions. An artificial slot was introduced into them all and to compare the rates they have all been drawn with a zero length of ½ in. The first item of interest is that the rate of crack propagation has relatively small scatter for a given design. The efficiency of the actual butt joint in stopping cracks is well demonstrated as is the advantage of a butt joint near the spar itself. By careful detail design the web life can be increased several-fold with little weight penalty.

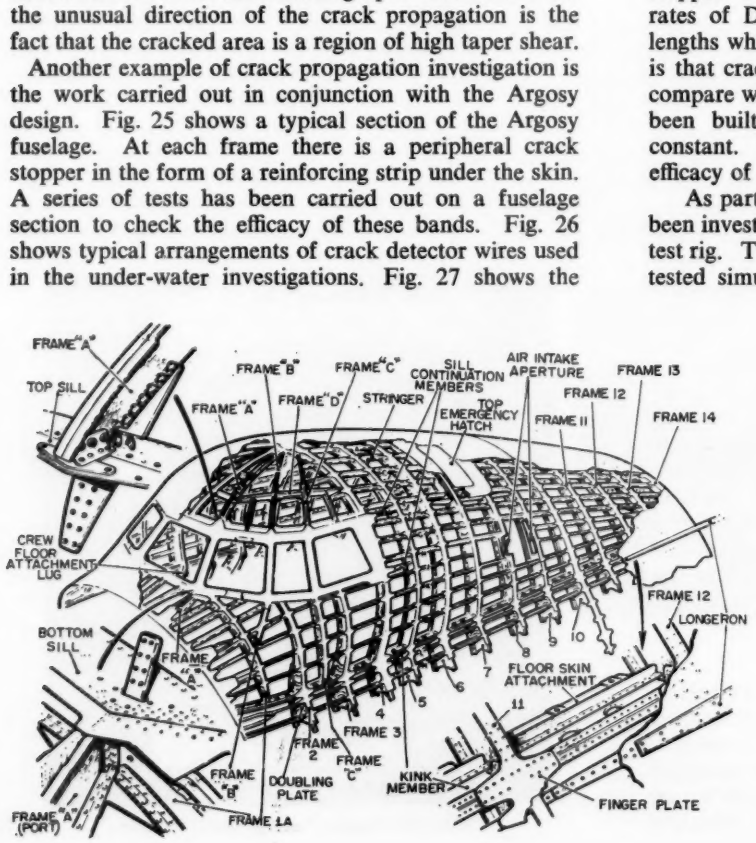


FIGURE 25. Argosy fuselage design.

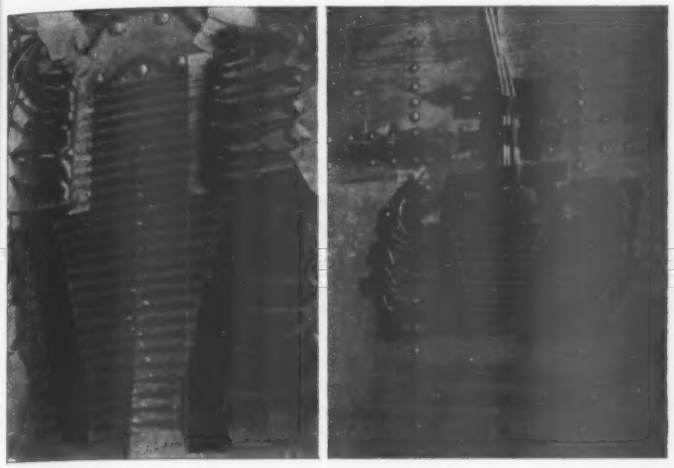


FIGURE 26 (*left*). Crack detector wires.

FIGURE 27 (*below*). Fuselage crack propagation rates. Cracks E and F are in D.T.D.546; all others are in D.T.D.710.

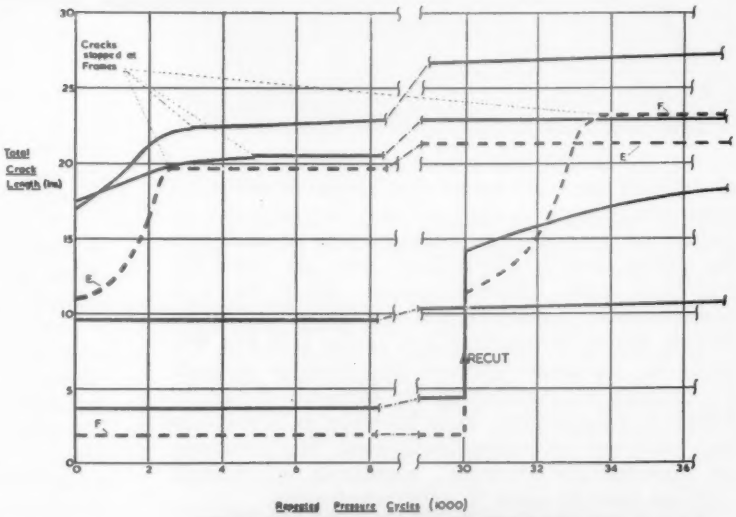


Figure 30 gives some initial work into residual static strength. The bottom curve gives the mean from test results on crack propagation rates on Redux panel test specimens. These show the remaining area after cycling from an initial crack of 4 per cent. The cycles are 10 ft./sec. gust cycles but as the specimens are not wide they are pessimistic. As this work is to give inspection times it should be noted that a year's flying would represent 60,000 cycles which would give about 12 per cent loss in area. However, it is the top curve which is the most important. Here allowable residual static strength is plotted against remaining area and the rapid fall off due to the notch effect is noticeable. The top line shows the theoretical based on net area and the other curves test evidence. Again the specimens were fairly narrow and width is obviously important but the results achieved fall on a well-defined line. The bottom results are actual fatigue results while the higher values are simple static tests with cut notches. The results show that 12 per cent lost in area gives about 50 per cent of the uncut strength. As in general a tension flange has a basic R.F. of about 1·3 from stress level conditions, the remaining strength will be about 67 per cent of design, *i.e.* meet A.R.B. fail safe requirements. But it is clearly early days in this work and we must attempt to improve the residual strength of structures and this type of testing will continue to be invaluable.

9. Notes on Test Technique

All tests should be arranged to obtain the maximum value for the expenditure involved. The cost of testing lies in the specimen and rig assembly and under no circumstances should the actual testing be rushed. As regards the specimen it must be as representative of the aircraft as possible especially as regards manufacturing technique, joint assembly and so on. One cannot stress too highly the importance of having the specimens built

FIGURE 28. Fail safe web test rig.

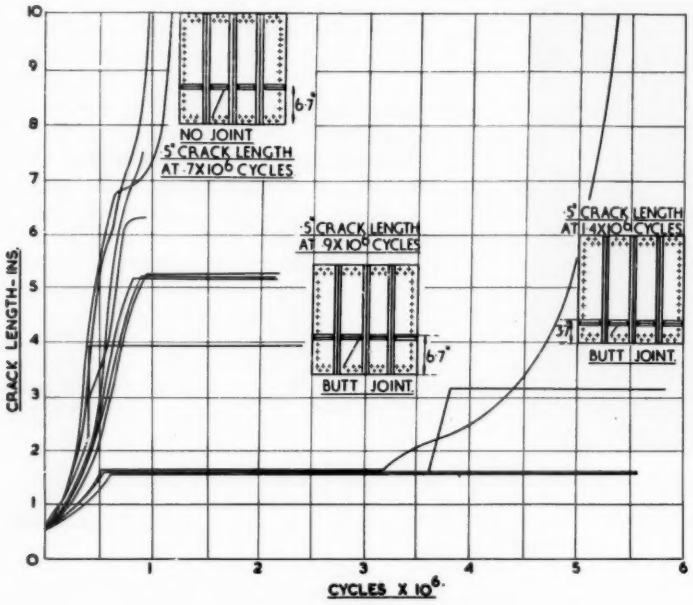


FIGURE 29. Fail safe web design.

by standard operatives rather than special experimental fitters when fatigue is the criterion. The applied load pattern and inspectional technique must be set out to find the highest limit of serviceability, not just prove the theoretical value. Instrumentation should be very comprehensive. Visual records are important, strain gauging should be discriminatory to cut cost and new measuring techniques evolved, photographic methods are valuable for establishing the sequence of failure, deflection measurements must be taken as a guide to limit loads and aeroelastic background. If all readings are automatically recorded, monitoring of selected locations must be made to anticipate trouble.

Two details which should be watched are functioning and environment. If a structure has any moving parts these should be functioned under load and with their supporting structure deflected. The environmental problem is rarely catered for. All test specimens are in good condition—with few scratches or dents and without any corrosion. I feel some cognisance of operating accidental damage and corrosion should be put in test specimens to give realistic results.

There is considerable evidence that testing should be continued up to the ultimate loads. Evidence to support this was given in the figures for aircraft structures and I consider this approach should be used throughout the aircraft, even on control systems. Fig. 31 shows a control rod failure on a complete fuselage test specimen between proof and ultimate load.

10. Full Scale Testing

The problem of full scale testing merits discussion on account of the time and effort consumed. Figs. 32 and 33 show tank test work proceeding on the Argosy. Any reduction in work in this field seems desirable.

The traditional history of structural testing in England has been the insistence on static testing to ultimate on all aircraft structures unless there is sufficient background to justify the waiving of this test, the introduction of water tests for checking pressurisation fatigue which was then amplified by the Comet experience to include complete aircraft for fatigue testing including water pressurisation cycles. This has led in England to the use of two complete aircraft specimens on certain projects, one static and one fatigue. This policy has been adopted on many aircraft including the Comet, the

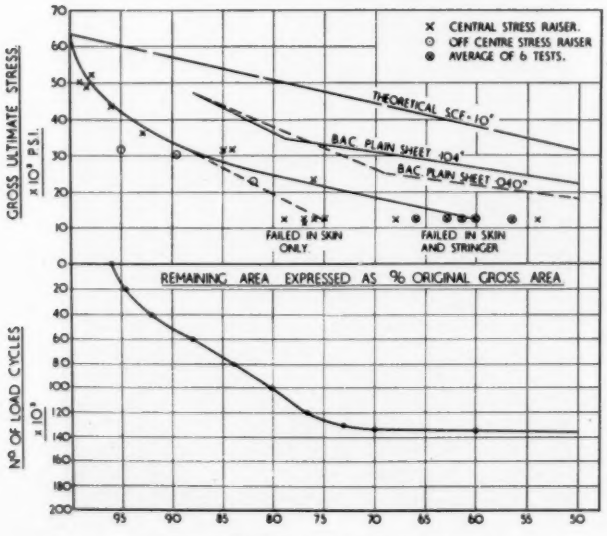

GROSS ULTIMATE STRESS x10³ P.S.I.
Nº OF LOAD CYCLES x10³
× CENTRAL STRESS RAISER.
○ OFF CENTRE STRESS RAISER
◉ AVERAGE OF 6 TESTS.
THEORETICAL SCF = 10
BAC PLAIN SHEET 104
BAC PLAIN SHEET 040
FAILED IN SKIN ONLY
FAILED IN SKIN AND STRINGER
REMAINING AREA EXPRESSED AS % ORIGINAL GROSS AREA

FIGURE 30. Residual static strength.

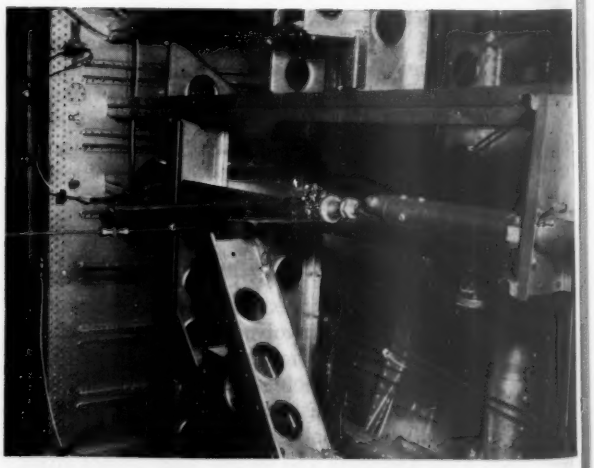


FIGURE 31. Control rod bracket failure.

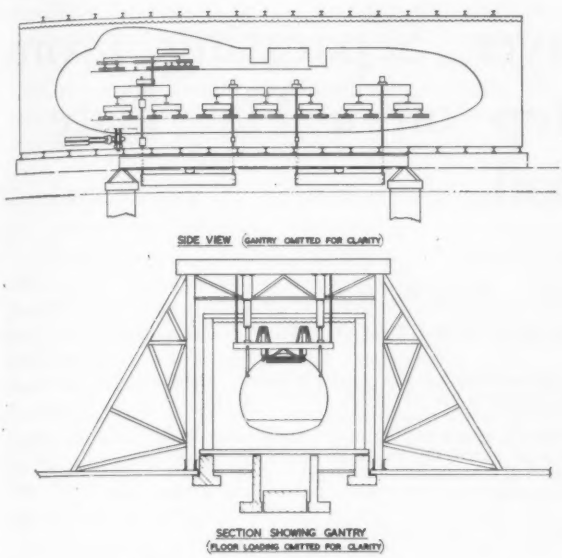


FIGURE 32. Fuselage in water tank.

FIGURE 33. Fuselage entering tank.

Britannia and currently, the Argosy and the Avro 748. The time has now come to consider whether this policy, which is extremely expensive, should not be reconsidered. The American civil policy is to test to only the proof load and then in many cases to actually put the test specimen back into airline service. American fatigue tests have been varied in their nature but a fair amount of military background is available on some of the civil transports whereas others intend to carry out a complete fatigue test in the conventional British manner. One feels that the American approach is extremely vulnerable in the static condition between proof and ultimate condition especially as the fail safe tests are probably only of a limited nature and the American fail safe requirements are in my opinion rather too low for safety. One feels, however, that the current British approach of two specimens is too expensive and needs modification. The obvious solution and the one which merits the most consideration is the policy of using one specimen with the following schedule of test.

First, brief tests with both pressurisation and wing loads with a possibility of a limited number of pressure reversals to cover prototype flying. Secondly, fatigue tests under a full fatigue spectrum up to about twice the safe life required. Thirdly, ultimate tests in conjunction with suitable fail safe tests to justify reduced safety factor of 2 on life. The main criticism of this approach is the time factor in that it is essential to speed up the tests since the critical ultimate test upon which the C. of A. is dependent is held to the very end, whereas the present policy of two specimens enables you to carry on the fatigue testing after the C. of A. date. However, there are many people who consider that even this one specimen approach is too expensive for today's current fail safe design and it is considered that if the principles upon which the aircraft is designed are proved by detailed tests or tests on a fairly large specimen that there is no need for carrying out a full scale test at all. The danger in only testing details is that there is no automatic check that the drawing office have incorporated the essential principles throughout and therefore it is possible that the fail safe characteristics obtained in a section of the monocoque are no longer certain in the cabin region or in cut-out regions. Secondly, testing has always shown that it is the subsidiary structure that gives the most trouble from a maintenance and possibly incipient ultimate failure standpoint and therefore it is essential that a complete aircraft is tested at some time or other. One feels that the history of structural design in England at this stage does not justify abandonment completely of testing full scale aircraft.

On a purely personal and somewhat political note I consider that one solution would be for the Ministry of Aviation to pay for the specimens and testing and make the results available to industry as a whole. In this way the development costs to the manufacturer would be reduced and the Aircraft Industry would benefit throughout by the spread of further knowledge.

ACKNOWLEDGMENTS

I would like to state that the views expressed throughout the paper are personal and do not necessarily represent those of Sir W. G. Armstrong Whitworth Aircraft Ltd., to whom I am grateful for permission to publish this paper. I would like to thank my associates in the Industry for their suggestions, especially the A.W.A. staff who have assisted in this paper.

Some Experiments on the Reattachment of a Laminar Boundary Layer Separating From a Rearward Facing Step on a Flat Plate Aerofoil

by

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SUMMARY: The results of experiments on the reattachment of a laminar boundary layer, separating from a rearward facing step in a flat plate aerofoil, are correlated with the properties of the short leading edge bubble which forms on thin aerofoils near the stall.

The experiments, comprising pressure measurements, Pitot explorations, liquid film and smoke studies, indicate that for all Reynolds numbers* above the value given by the Owen-Klanfer criterion the reattachment is turbulent behind a stationary air reverse flow vortex bubble. It is also found that the reattachment is laminar for Reynolds numbers below the critical, which further supports Crabtree's interpretation of the Owen-Klanfer criterion in terms of the condition for the growth of turbulent bursts.

NOTATION

C_{p1}	pressure coefficient immediately behind laminar separation point (*see* Fig. 3)
C_{p2}	pressure coefficient at reattachment (*see* Fig. 3)
h	step height
l_b	bubble length
l	distance behind step
L	distance from leading edge to step
u	local velocity in boundary layer
U	local velocity at edge of boundary layer
R_δ	Reynolds number based on boundary layer displacement thickness and local free stream velocity
σ	pressure recovery coefficient $= \dfrac{C_{p2}-C_{p1}}{1-C_{p1}}$

1. Introduction

The flow over thin aerofoils at incidences near the stall is characterised by the presence of a "bubble" formed by separation of the laminar boundary layer close to the leading edge and its reattachment as a turbulent boundary layer. It appears that there are two types of bubble which have, in the past, been called "short" and "long". The "short" bubble is usually very small ($\simeq$one per cent of aerofoil chord), its size varies with Reynolds number, and it has almost no effect on the suction peak ahead of the laminar separation point with increasing incidence. The long bubble, however, causes a fall of the suction peak with increasing incidence and expands rapidly towards the trailing edge as the stall is approached. It also appears that the size of the long bubble is influenced primarily by aerofoil shape and incidence, the effect of Reynolds number being small.

Thin aerofoil stall† begins with the formation of the first type, namely the short bubble, and a point is reached when this bubble suddenly "bursts." The long bubble then appears and as incidence is increased it increases in size until the reattachment point moves behind the trailing edge. Since the size of the long bubble is primarily dependent on incidence it is evident that if the short bubble persists up to high incidences the long bubble will be unable to reattach. Complete separation then occurs with the sudden loss of lift which has been observed on some aerofoils. It is therefore apparent that the breakdown of the short bubble is a possible cause of thin aerofoil stall.

Owen and Klanfer[1] postulated a simple criterion which determines which type of bubble is likely to appear for a given boundary layer Reynolds number at separation. The criterion states that if R_δ (where δ is the boundary layer displacement thickness at separation) is greater than 400-500 the bubble will be short, but if R_δ falls below this range the bubble becomes long. Crabtree[2] has, on the basis of this criterion, advanced a physical explanation for the above type of stall. This explanation suggests that the importance of this critical value of R_δ lies in the fact that it determines the limit below which turbulent bursts fail to grow in the separated layer and so to provoke transition. A short bubble with turbulent re-attachment supporting an appreciable pressure recovery is possible for conditions in which R_δ exceeds this critical and the pressure recovery coefficient $\sigma=(C_{p2}-C_{p1})/(1-C_{p1})$ is below a certain maximum value. When this maximum is reached the bubble bursts; the leading edge suction peak collapses and the resulting redistribution of nose pressures causes the value of R_δ to fall below the critical, whence a long bubble is formed. The author's earlier paper[3] confirmed that σ does attain a maximum near 0·36.

The present paper gives the results of some experiments on the reattachment of a boundary layer that separates as a laminar layer from a rearward facing step in a flat plate set at zero incidence in two-

*Reynolds numbers are based on boundary layer displacement thickness at separation.

†"Thin aerofoil stall" is here used in the sense defined in Ref. 3; in Ref. 2 it is termed 'leading edge stall".

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dimensional low-speed flow. The experiments show that the reattachment can be laminar or turbulent depending on the value of R_s, with the bubble thus displaying a similarity to the leading edge bubbles on aerofoils. The results are discussed in relation to Crabtree's interpretation of the Owen-Klanfer criterion.

2. Experiments on a Rearward Facing Step

2.1. OBJECT OF TESTS

The presence of a rearward facing step in a flat plate will induce separation of the boundary layer. The prevailing pressure gradients in front of and behind the step can be made small by setting the flat plate at zero incidence, thus enabling the boundary layer to reattach itself behind the step to form a closed bubble. If the boundary layer is laminar at separation this bubble might simulate the "short" (or "long") bubble found on the leading edge of a thin aerofoil. This approach is introduced because it has the following advantages over experiments on aerofoils.

(*a*) The size of the bubble and the boundary layer Reynolds number at separation can be made large in comparison with the size of the test model.

(*b*) The prevailing pressure gradients can be made small in comparison to those present on an aerofoil.

It should, however, be remembered that there is still a finite pressure rise in potential flow associated with the stream tube expansion from the step, though this should be less severe than on aerofoils. The smaller pressure gradients would promote reattachment. This follows from the argument advanced in the author's previous paper (*see* Fig. 1(*a*) and Discussion, Ref. 3) which says that if there is a maximum pressure recovery available in the reattachment process, the bubble can reattach more easily if the required pressure rise is small. If, further, the required pressure rise is very small, it might even be possible to obtain a laminar reattachment since little pressure recovery would be required from the reattachment process.

It seems reasonable to use a step size which is the same order as the boundary layer thickness at the step. Having thus formulated the model the aim of the tests was to establish the nature of the flow behind the step, its influence on the pressure distribution and the effects of boundary layer Reynolds number at the step on the reattachment process. The results might then be used to obtain information on thin aerofoil stalling behaviour, and in particular the Owen-Klanfer criterion.

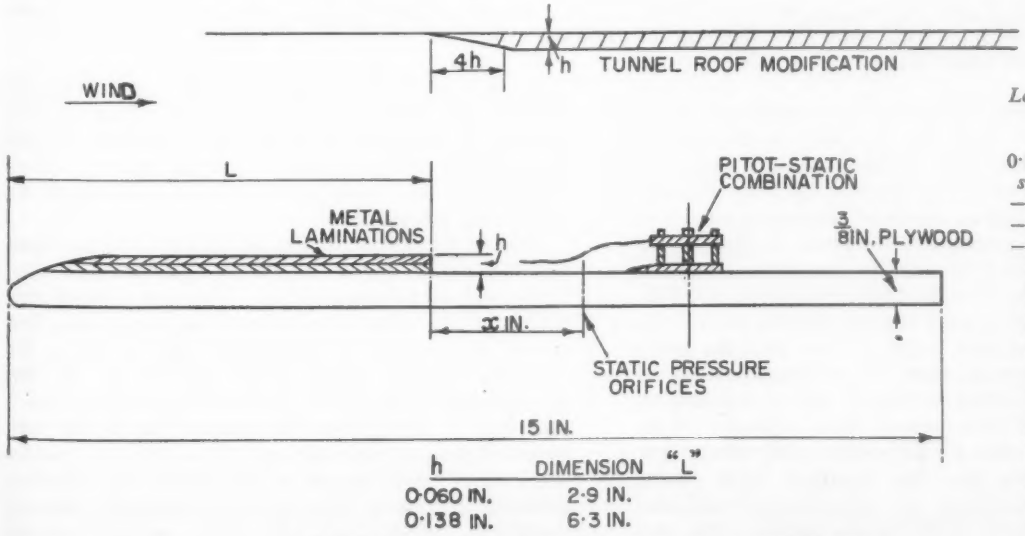
2.2. APPARATUS

The tests were conducted in the 1 ft. × 1 ft. low speed wind tunnel at the University of Southampton. This had a 4:1 contraction ratio and a set of turbulence reducing screens at the entrance to the contraction section. The model consisted of a 15 in. long plate aerofoil with a faired leading edge and with a rearward facing step which is illustrated in Fig. 1. Metal laminations were used as indicated in this diagram to obtain step sizes $h=0{\cdot}060$ in. and 0·138 in. with corresponding lengths ahead of the step of 2·9 in. and 6·3 in., which gives a constant ratio for L/h. Static pressure orifices were drilled in the surface at the stations indicated.

The instrumentation consisted of a Pitot-static combination for traversing the boundary layer and a Cambridge precision manometer for the pressure measurements. The Pitot-static combination had long probes designed to reduce interference to the minimum.

2.3. DESCRIPTION OF TESTS

The model was set to give pressure readings corresponding to $C_p=0$ for the orifices ahead of the step and for those orifices some distance behind the reattachment region. This was done with the aid of a wooden insert in the roof of the tunnel as indicated in Fig. 1. The boundary layer was laminar at separation and its



Location x in. of static pressure orifices	
0·138 *in. step*	0·060 *in. step*
−2·34	−0·96
−1·24	−0·03
−0·28	0·03
0·03	0·49
0·18	0·39
0·36	0·93
0·60	1·24
0·75	1·75
1·04	2·38
1·34	2·90
1·77	3·50
2·32	4·50
2·84	5·76
3·46	8·00
4·04	
4·96	

FIGURE 1. Diagram of model for flow over step experiments.

reattachment was studied at wind speeds from 32 ft./sec. to 50 ft./sec. giving a Reynolds number (R_δ) range from 338 to 750 with the two step sizes.

The tests consisted of boundary layer traverses, pressure measurements and liquid film (lampblack in paraffin) studies of the flow, a few inches behind the step. These tests were supplemented by a few smoke studies, with smoke introduced at a very low velocity behind the step.

3. Results

Figure 2 shows a comparison of the experimental methods for a test Reynolds number (R_δ) of 620 and on the 0·138 in. step. The lampblack pattern and the shape of the pressure distribution leads to the deduction of the physical structure of the flow. A strong accumulation of liquid behind the step corresponds with a region of constant negative pressure, and this suggests a zone of almost stationary air underneath a separated laminar boundary layer. The half herring bone appearance behind the accumulation, together with a rapid pressure rise suggests a strong reverse flow vortex underneath the separated layer which has then become turbulent. The steep pressure rise associated with the reverse flow vortex can be compared with the characteristic pressure recovery on a thin aerofoil bubble, where Crabtree's arguments in Ref. 2 suggest that transition is at, or near, the point where the pressure recovery starts. It can be tentatively concluded therefore that transition is located by the start of the pressure rise. The pressure becomes positive or " overshoots " the $C_p=0$ level and then more gradually returns from C_{p_2} to zero. The end of this pressure rise roughly corresponds to the appearance of a mottled pattern and suggests that the turbulent boundary layer has reattached to the surface. The fall of pressure to zero indicates that after reattachment the pressures follow the inviscid flow values which arise from contraction of the stream tubes while the boundary layer develops the characteristic turbulent shape profile. The structure of the flow is, in fact, similar to that within a short leading edge bubble with the characteristic reverse flow vortex which provides a pressure recovery at reattachment.

Pressure distributions for each step height are shown in Figs. 3 and 4. With the exception of the plot for $R_\delta=338$, all show similar features, namely regions of constant negative pressure, followed by steep pressure rises and a more gradual return of pressure to zero. The experimental points behind reattachment lie almost on a common curve which is probably the inviscid pressure fall behind the step. The $R_\delta=338$ curve for the 0·060 in. step in Fig. 4 shows a much weaker pressure rise and the overshoot pressure peak is much lower than the peaks for the higher Reynolds numbers. A comparison of the boundary layer profiles in Figs. 5 and 6 suggests an explanation to this phenomenon. Fig. 5 is typical of the boundary layer profiles for all pressure plots which show the steep pressure rise; the boundary layer profile appears to be turbulent at reattachment, and this suggests a turbulent reattachment bubble. On this diagram the end of the constant pressure and pressure

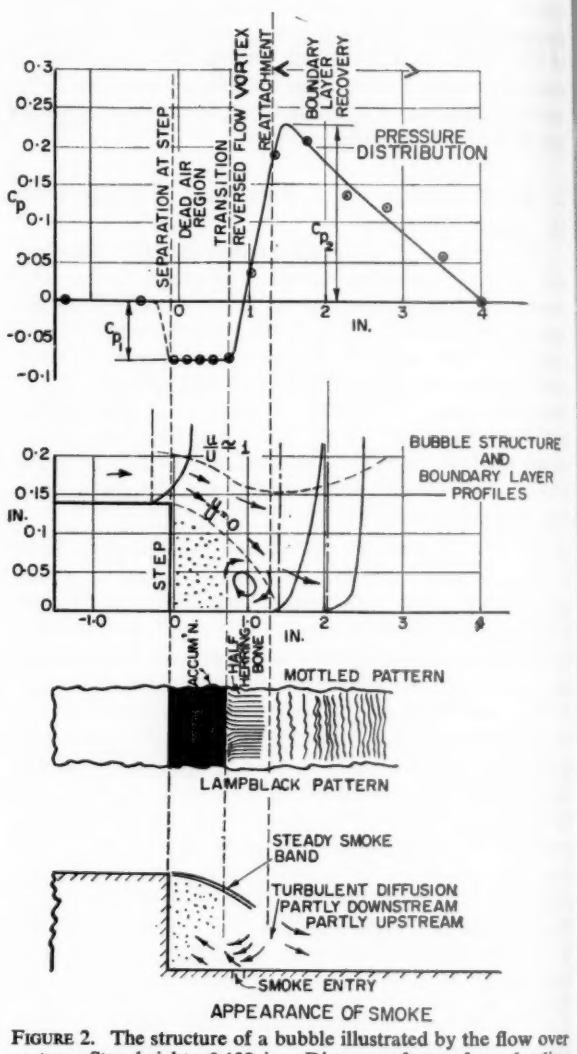


FIGURE 2. The structure of a bubble illustrated by the flow over a step. Step height = 0·138 in. Distance of step from leading edge = 6·3 in. $U_0=50$ ft./sec. $R_\delta=620$.

recovery regions from Fig. 3 at the same Reynolds number are shown. It is apparent that there is little increase in boundary layer thickness between the step and the end of the constant pressure recovery. This supports the conclusion that transition is located by the start of the pressure rise.

Figure 6 displays profiles which are more consistent with the notion that the boundary layer has reattached while still in a laminar state. Laminar reattachment would greatly reduce the strength of the reverse flow vortex at the rear of the bubble and so reduce the pressure recovery, which would account for the low pressure peak at $R_\delta=338$. Instead of comparing Figs. 3 and 4 at a given Reynolds number, Fig. 7 has been prepared from these diagrams. This shows curves for the ratio of bubble length to step height, the overshoot pressure coefficient C_{p_2} and the pressure recovery coefficient $\sigma=(C_{p_2}-C_{p_1})/(1-C_{p_1})$ plotted against Reynolds number.

It was assumed that the bubble length l_b is represented by the distance of the pressure peak behind the step. These results indicate that for all turbulent reattachment cases the bubble length decreases linearly with increasing Reynolds number while C_{p2} and σ remain almost constant, the latter at a value well below the critical maximum value of 0·36. The curves also indicate a remarkable independence of step height, and the large increase of bubble length with fall of C_{p2} and σ for the $R_\delta = 338$ case.

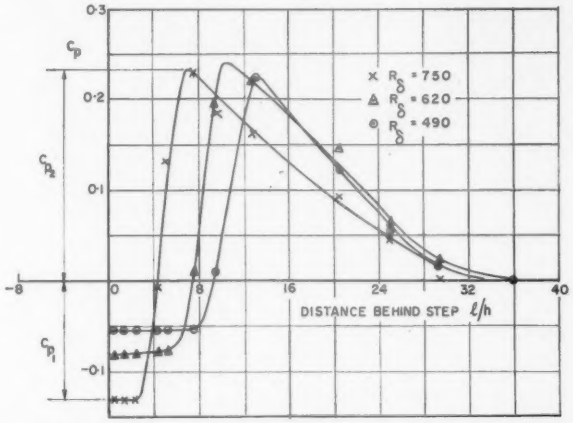


FIGURE 3 (*right*). Pressure distributions behind 0·138 in. step.

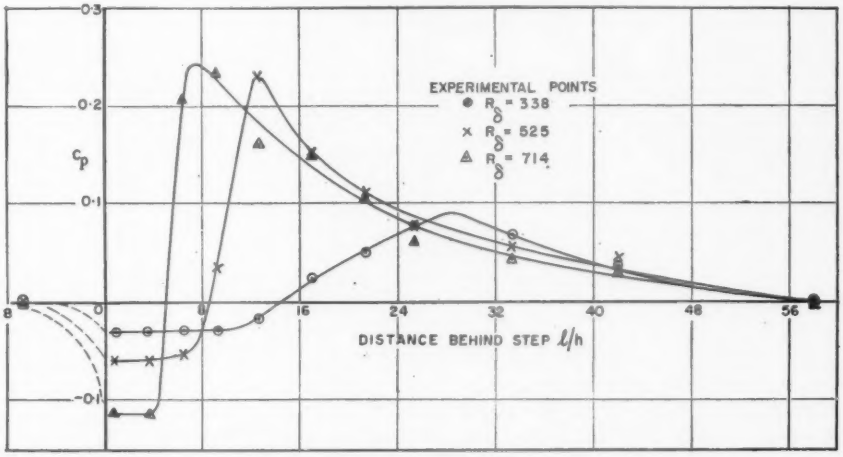


FIGURE 4 (*right*). Pressure distributions behind 0·060 in. step.

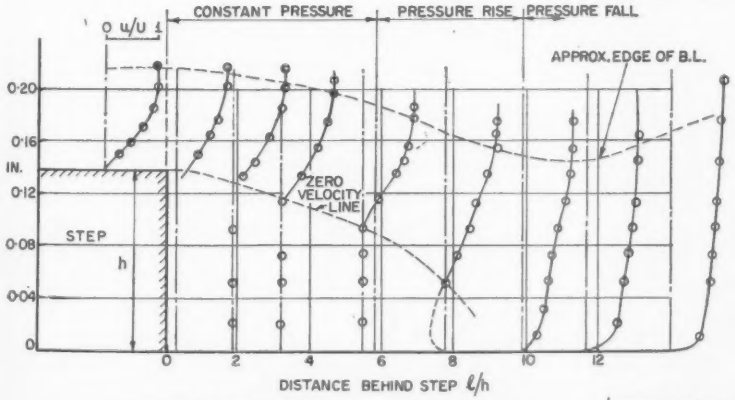


FIGURE 5 (*left*). Boundary layer velocity profiles behind 0·138 in. step. $R_\delta = 620$.

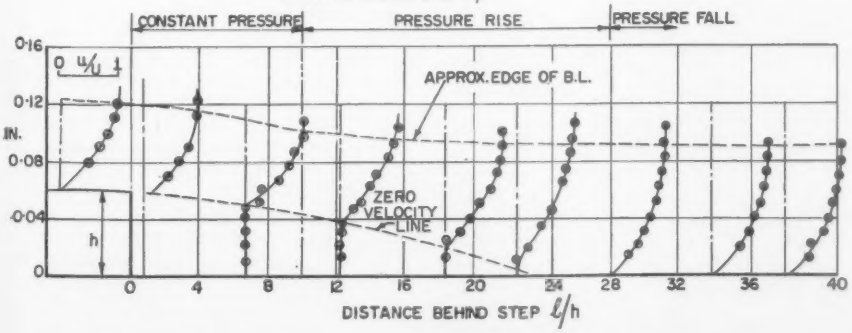


FIGURE 6 (*left*). Boundary layer velocity profiles behind 0·060 in. step. $R_\delta = 338$.

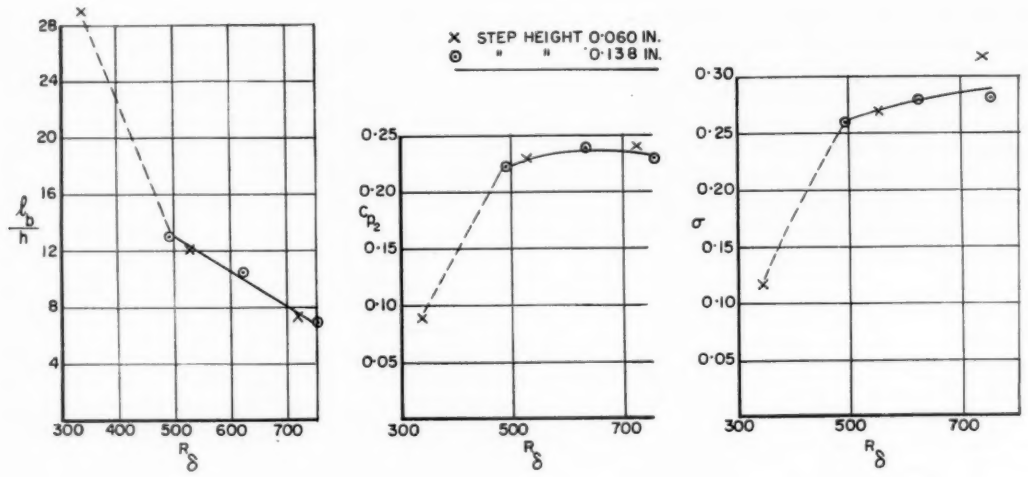


FIGURE 7. Variation of l_b/h, C_{p_2} and σ with R_δ.

4. Discussion

The experiments have shown that the separated laminar boundary layer reattaches behind a stationary air reverse flow vortex bubble. The value of σ is below the maximum value of 0·36 postulated for reattachment but of the same order as that encountered on unstalled aerofoils with short bubbles. This suggests that the test results can be correlated with short bubble behaviour on a thin aerofoil at incidence. Any such correlation would be further justified as the bubble behaviour is virtually independent of step height*, and the change of bubble length with Reynolds number.

Let us now compare the results with the Owen-Klanfer criterion. Figs. 4, 6 and 7 have shown the marked change in the bubble behaviour for the lowest Reynolds number tested, namely the laminar reattachment and the lower value of σ. The boundary layer has thickened at reattachment but even so the displacement thickness Reynolds number is still about 500, and the profiles are those of a laminar boundary layer rather than a turbulent one. This then is evidence of a much delayed transition when the separation Reynolds number is below the Owen-Klanfer critical, supporting Crabtree's interpretation of this critical value.

In conclusion it can be said that the step bubble simulates the following thin aerofoil short bubble characteristics.

(*a*) The stationary air region below a laminar boundary layer which increases only slowly in thickness, and the dependence of the length of this region on Reynolds number.

(*b*) The turbulent reattachment mechanism at higher values of R_δ.

The evidence presented supports the conclusion that short bubble flow cannot exist if R_δ is less than about 500. The experimental set-up (flow separation from a step) has enabled this to be done without the distracting complications which arise on aerofoils because of the need for the flow patterns in the presence of strong adverse pressure gradients to re-adjust themselves when the pressure recovery factor would otherwise exceed the critical value of about 0·36.

*This is so provided h/L is constant.

5. Conclusions

Experiments on a rearward facing step in a flat plate aerofoil show that the laminar boundary layer usually reattaches as a turbulent layer to form a bubble behind the step. The following conclusions have been drawn about the behaviour of this bubble.

(*i*) The bubble simulates the short bubble found on the leading edge of thin aerofoils at high incidence without the complications associated with strong adverse pressure gradients.

(*ii*) Transition is delayed as the Reynolds number falls below the critical. This is consistent with Owen and Klanfer's criterion relating to the formation of long and short bubbles, and supports Crabtree's interpretation of that criterion.

(*iii*) The pressure recovery coefficient σ and C_{p_2} attain nearly constant values for turbulent reattachment bubbles behind the step, and the bubble behaviour is independent of step height provided L/h is constant.

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Creep-Stress Analysis of Thin-Walled Structures*

by

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1. Introduction

In the present era of supersonic and hypersonic flight, structural analysts are well aware of the new problems that arise from the exposure of aircraft and missile structures to elevated temperatures. The task of solving these problems is in its early stage of development. Since the problems are diversified in nature, a different method of solution has to be developed in each case. It is probable that in some cases radically new techniques may have to be devised. Until such time, the structural designer is forced to take a realistic view and use satisfactory extensions of existing methods of analysis.

The particular high temperature effect in the structure under investigation determines the necessary extensions of a current theory. For instance, the analysis of a structure for creep stresses requires a line of approach different from the analysis of the same structure for thermal stresses caused by temperature gradients. Moreover, in the investigations of two types of structures subject to the same effects (*e.g.* creep) the method of attack may have to be modified. Finally, where the mathematics of a problem becomes intractable, the only recourse may be to obtain approximate solutions.

With the foregoing considerations in view, this paper presents some methods of stress analysis of thin-walled structures subject to creep. It is assumed that the structures considered are maintained at a uniform temperature so that no thermal stresses are present. A further assumption is made that the elastic deformations are negligible in comparison with creep deformations. Under these assumptions, the creep problem of thin-walled structures is first reduced to analogous problems of the same structures made of nonlinear elastic materials. The nonlinear stress analyses are then carried out with particular reference to structures with closed and open cross sections and subjected to pure torsion, bending and shear.

NOTATION

A, A_1, A_2, A_3, A_s	areas
C, E	material constants
$I_n, I_{x,n}, I_{y,n}, I_{x',n}, I_{y',n}$	definite integrals of geometry and material
J_2	stress deviation invariant
$M, M_x, M_y, M_{x'}, M_{y'}$	bending moments
$Q_n, Q_{x,n}, Q_{y,n}, Q_{x',n}, Q_{y',n}$	indefinite integrals of geometry and material
R	radius
$S, S_1, S_2, S_{21}, S_{31}, S_{32}$	lengths
T	torque
$U_c, \overline{U}_c$	complimentary energies
$V, V_x, V_y, V_{x'}, V_{y'}$	shear forces
a, b, d, e', e_β, h	lengths
k, m, n, p	material constants
q, q_0, q_1, q_2, q_3	shear flows
s, s_1, s_2, s_3	lengths
s_{ij}	stress deviation tensor
$t, t_1, t_2, t_3, t_{21}, t_{31}, t_{32}$	thicknesses
u, v, w	displacements
$x, y, z; x', y', z$	co-ordinates
Θ	time
Ω, λ	material constants
$\alpha, \alpha_0, \beta, \phi$	angles
α_{21}, α_{31}	shear flow ratios
δ	length
δ_{ij}	Kronecker delta
ε	strain
ε_{ij}	strain tensor
η, η_0, η_1	dimensionless shear flows
θ	angle of twist
κ	curvature
μ, ξ	length ratios
σ, τ	stresses
$\bar{\sigma}, \bar{\sigma}_s, \bar{\tau}, \bar{\tau}_s$	dimensionless stresses
σ_{ij}	stress tensor

2. Creep Law and its Analogue

There exist different formulations of laws relating creep deformation to its various governing parameters. Based on Prager's[1] general stress-strain law for incompressible isotropic elastic material, Hoff[2] proposed a comparatively simple law to account for creep in

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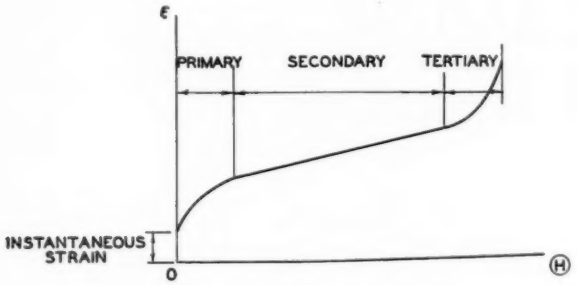

FIGURE 1. Typical creep curve.

structural materials subject to a triaxial state of stress. This law may be written

$$\varepsilon_{ij}=CJ_2^{\,m}s_{ij}\Theta^{1/p}, \quad . \quad . \quad (1)$$

where Θ represents time, ε_{ij} is the strain tensor, s_{ij} is the stress deviation tensor defined by*

$$s_{ij}=\sigma_{ij}-(1/3)\,\sigma_{kk}\delta_{ij}, \quad . \quad . \quad (2)$$

and J_2 is the second invariant of the stress deviation tensor

$$J_2=\tfrac{1}{2}\,s_{ij}s_{ij}. \quad . \quad . \quad (3)$$

The material parameters C, m, and p in equation (1) must be obtained from constant temperature creep tests.

Although various creep data from experiments on specimens subject to triaxial stresses have been compiled, they show considerable scatter for the evaluation of the constants C and m. However, Hoff[2] has proposed that these constants may be obtained from creep experiments in simple tension. In such a case, the general equation (1) assumes the form

$$\varepsilon=(\sigma/\lambda)^n\Theta^{1/p}, \quad . \quad . \quad . \quad (4)$$

where the material constants λ and n may now be deduced from tensile creep tests at constant temperature. Further, if n is assumed an odd integer, equation (4) applies equally in tension and compression. The condition that equation (1) reduces to equation (4) then provides the relations

$$\left.\begin{aligned} 2m&=n-1,\\ 2C&=\frac{3^{(n+1)/2}}{\lambda^n}. \end{aligned}\right\} \quad . \quad . \quad (5)$$

With these constants, equation (1) may be used as the basis for the creep analysis of structures subject to stresses in more than one dimension.

A typical variation of strain with time (equation (4)) is illustrated in Fig. 1. The constants λ, n, and p in the primary phase for 75S-T6 aluminium alloy are shown plotted as a function of temperature in Fig. 2. These plots were obtained on the basis of data given in Refs. 3 and 4. Since the creep curve is approximately a straight

*Latin letters used as subscripts take on the values x, y or z. A repeated subscript is to be summed from x to z. The Kronecker delta δ_{ij} is equal to one if $i=j$, 0 if $i\neq j$. Thus equation (2) stands for three relations of the type

$$s_{xx}=\sigma_{xx}-(1/3)\,(\sigma_{xx}+\sigma_{yy}+\sigma_{zz})=(1/3)\,(2\sigma_{xx}-\sigma_{yy}-\sigma_{zz})$$

and six symmetric relations of the type

$$s_{xy}=\sigma_{xy}.$$

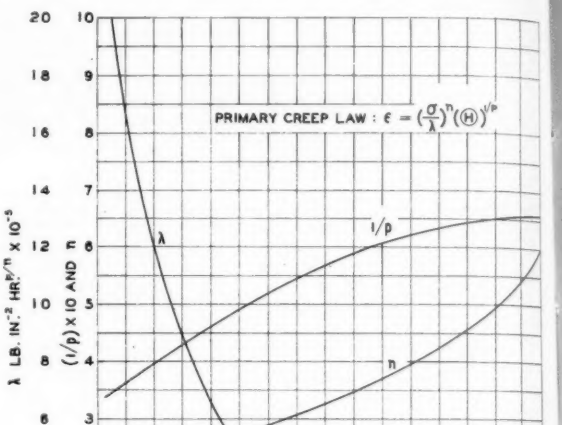

FIGURE 2. Material parameters for 75S-T6 aluminium alloy.

line in the secondary phase (Fig. 1), the constant p assumes the value unity in this phase and the corresponding values of λ and n denoted by the symbols Ω and k, respectively, are shown as a function of temperature in Fig. 3 for the same material. The variation of Young's modulus E with temperature is also shown in this figure.

Hoff[2] has shown that the creep problem of a structure governed by equation (1) may be transformed to an analogous nonlinear elastic problem. If it is

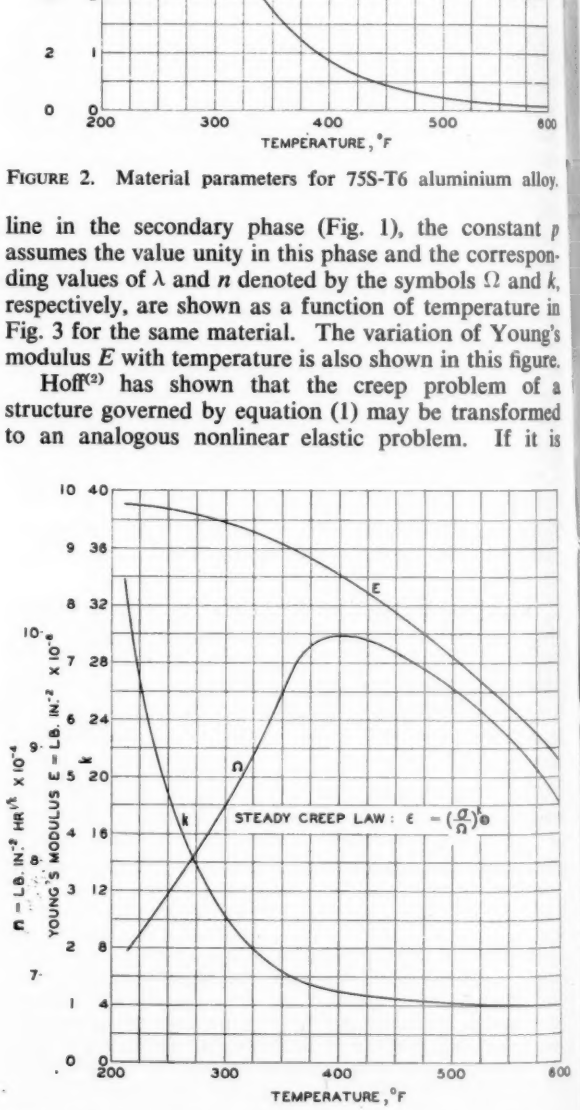

FIGURE 3. Material parameters for 75S-T6 aluminium alloy.

assumed that the elastic problem is based on the nonlinear law

$$\varepsilon_{ij} = CJ_2^{m} s_{ij} \quad . \quad . \quad . \quad (6)$$

and the relation between the elastic and creep strains is

$$(\varepsilon_{ij})_{cr} = \varepsilon_{ij}\, \Theta^{1/p}, \quad . \quad . \quad (7)$$

then the two problems are mathematically identical. Thus, the same structure subject to the same loads is to be considered, but the structure is now to be governed by equation (6) and the velocities are to be obtained from the relation

$$u^{p} = \frac{d}{d\Theta}\,[u_{cr}{}^{p}]\,. \quad . \quad . \quad . \quad (8)$$

Here, u and u_{cr} represent, respectively, the elastic and creep displacements. If $p=1$, the analog applies to the secondary or steady phase and again λ and n assume, respectively, the values Ω and k.

The elastic analogue has previously been used in the solution of secondary creep problems[5, 6, 7]. Once the nonlinear elastic problem is solved, the stresses in the creep problem are immediately known. The creep strains are readily obtained by the use of equation (7). The remainder of the paper will discuss the analysis of some thin-walled structures whose material is governed by equation (6)[8, 9, 10, 11].

3. Multi-Cellular Torque Box

Thin-walled structures often form major components of aircraft and missile structures. For instance, wings of aircraft are usually in the nature of beams of cellular sections. The behaviour of such beams when subjected to applied torques is an important design consideration. Typical beams are analysed in this section and illustrated by application to a few examples.

When a cell having the cross section shown in Fig. 4 is subjected to simple torsion (St. Venant torsion), the shear strain is given by

$$2\varepsilon_{sz} = (\partial v/\partial z + \partial w/\partial s). \quad . \quad . \quad (9)$$

Here, v and w are, respectively, the displacement components of a generic point in the s and z directions. By virtue of the stress-strain law (6) and the relation (5), it follows that

$$2C\tau_{sz}{}^{n} = 2\varepsilon_{sz} = (\partial v/\partial z + \partial w/\partial s). \quad . \quad (10)$$

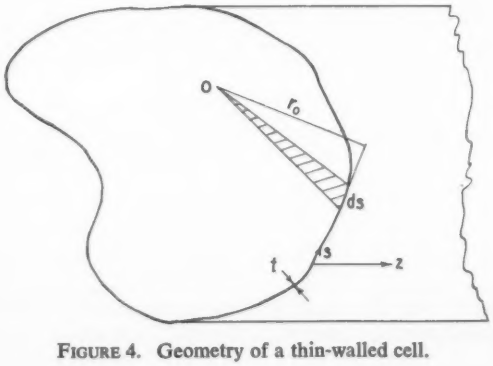

FIGURE 4. Geometry of a thin-walled cell.

If θ is the angle of twist per unit length of the beam, it may be verified from Fig. 4 that $\partial v/\partial z = r_0\theta$; r_0 being the perpendicular distance from any point 0 to the tangent at the line segment considered. Therefore, equation (10) simplifies to

$$2C \oint \tau_{sz}{}^{n} ds = \theta \oint r_0 ds. \quad . \quad . \quad (11)$$

It can be shown that $\oint r_0 ds$ is twice the area A enclosed by the cell. It follows then, that

$$\theta = (C/A) \oint (q/t)^{n} ds, \quad . \quad . \quad (12)$$

where $q = \tau t$ is the shear flow and is assumed constant as in linear elastic theory. Equation (12) represents the relation between the rotation per unit length and the shear stress for any closed section.

For the case of a single cell, equilibrium requires that torque $T = 2qA$. Therefore, equation (12) simplifies to

$$\theta = (C/A)(T/2A)^{n} \oint ds/t^{n}. \quad . \quad (13)$$

If in addition to the shear flow q, the thickness t is assumed constant, equation (13) reduces to the simple relation

$$\theta = (CS/A)(T/2At)^{n}, \quad . \quad . \quad (14)$$

where $S = \oint ds$ is the developed length of the median line.

If the single cell considered is linearly elastic, *i.e.* $n=1$, equations (13) and (14) reduce to the classical Bredt's formulae. Now, these two equations are particular forms of the more general equation (12). This last equation may be considered a modified form of Bredt's formula. It will be used in the succeeding analyses of torque boxes of two and three cells as applicable to each individual cell.

In the cross section of the two-cell beam shown in Fig. 5, A_1 and A_2 are, respectively, the areas enclosed by the median lines of cells 1 and 2. The corresponding shear flows, for an applied torque T, are q_1 and q_2. As in linear elasticity, it is now assumed that both cells have the same angle of twist θ per unit length. Thus equation (12) may be used to compute θ for each cell and the results equated to yield the flow compatability condition

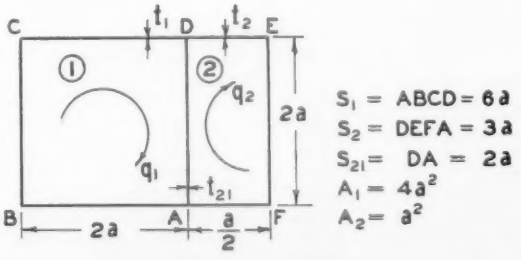

FIGURE 5. Two-cell torque box.

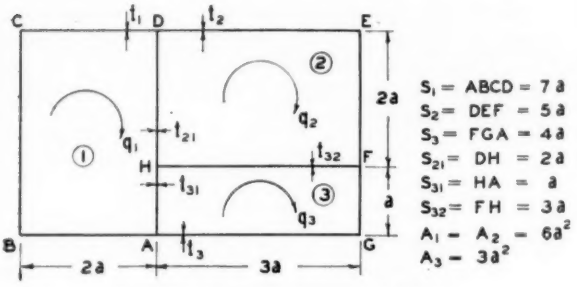

FIGURE 6. Three-cell torque box.

$$(A_2/A_1)\{(q_1/t_1)^n S_1+[(q_1-q_2)/t_{21}]^n S_{21}\} =(q_2/t_2)^n S_2-[(q_1-q_2)/t_{21}]^n S_{21}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

Further, the equilibrium of the applied torque and shear flow requires that

$$T=2q_1A_1+2q_2A_2. \quad . \quad . \quad (16)$$

For any given integer n, equation (15) may be solved for the ratio $\alpha_{21}=q_2/q_1$. Substitution of this value in equation (16) determines the shear flows q_1 and q_2. These values when substituted in equation (12) yield the torque-twist relation

$$\theta=C\,(S_1/A_1)\,\frac{1+[(1-\alpha_{21})/(t_{21}/t_1)]^n\,(S_{21}/S_1)}{[2A_1t_1\,(1+\alpha_{21}A_2/A_1)]^n}\,.\,T^n. \quad (17)$$

It may be verified that for $\alpha_{21}=0=A_{21}$, equation (15) vanishes and equations (16) and (17) reduce to those of the single cell case.

The preceding analysis of the two-cell section may be readily extended when a torque box having a three-cell section (Fig. 6) is considered. The analysis leads to two flow compatibility conditions similar to equation (15) and an equilibrium equation similar to equation (16) (*see* Ref. 11). The solution of these for the shear flows and substitution into equation (13) yields the torque-twist relation

$$\theta=C\,(S_1/A_1)\times \frac{1+[(1-\alpha_{21})/(t_{21}/t_1)]^n\,(S_{21}/S_1)+[(1-\alpha_{31})/(t_{31}/t_1)]^n\,(S_{31}/S_1)}{[2A_1t_1\,(1+\alpha_{21}A_2/A_1+\alpha_{31}A_3/A_1)]^n}T^n \quad . \quad . \quad . \quad (18)$$

where $\alpha_{21}=q_2/q_1$ and $\alpha_{31}=q_3/q_1$.

The above analyses indicate that when the torque box has N cells, there will be $N-1$ simultaneous non-linear flow compatability equations to determine the $N-1$ shear flow ratios. Substitution of these ratios in the equilibrium equation would again yield the shear flow q_1. The other shear flows simply follow as the products of q_1 and each of the ratios.

As illustrations of the analyses presented, two numerical examples will now be briefly considered. The first example concerns the two-cell torque box shown in Fig. 5. With the geometrical values shown in the figure, the flow compatibility equation (15) reduces to

$$5\,(1-\alpha_{21})^n-6\alpha_{21}{}^n+3=0. \quad . \quad . \quad (19)$$

The equilibrium equation (16) and the torque-twist relation (17) reduce, respectively, to

$$q_1=T/2a^2\,(4+\alpha_{21}), \quad . \quad . \quad (20)$$

$$\theta=(C/2a)\,.\,\frac{3+(1-\alpha_{21})^n}{[2\,(4+\alpha_{21})]^n}\,.\,\left(\frac{T}{a^2t_1}\right)^n \quad . \quad . \quad (21)$$

Results of the solution of equation (19) for α_{21} for assumed values of n are presented in the first two columns of Table I. The corresponding values of the shear flow q_1 are then computed from equation (20) and the shear flow values $q_2=\alpha_{21}q_1$ follow very simply. Column 3 in Table I denotes the ratios of the values of q_1 for any n to that when $n=1$. Column 4 denotes similar results for cell 2.

Examination of Table I shows that the increment in α_{21} is about 27 per cent for a variation in n from 1 to ∞. For this latter value of n, $q_1=q_2$ and there is no shear flow in the web. While the ratio $q_{1,n}/q_{1,1}$ varies only about 5 per cent in the range $n=1$ to $n\longrightarrow\infty$, the corresponding variation in $q_{2,n}/q_{2,1}$ is a substantially higher 30 per cent. As a further example, the three-cell torque box shown in Fig. 6 is considered for the geometrical values as indicated. The results are presented in Table II.

4. Bending of Thin-Walled Open Section Beams

The stringers most commonly used in aircraft practice have open cross sections and are made of thin gauge materials. The bending moments and shear forces acting on these stringers give rise to bending and shear stresses and corresponding deformations. The distribution of these stresses and deformations varies with the type of cross section considered. Although a relatively general analysis of an arbitrary cross section may be desirable, it is not feasible in view of the number of variables involved. Even when the arbitrariness of the open section is restricted by the assumption of an axis

TABLE I

SHEAR FLOW RATIOS FOR THE TWO-CELL EXAMPLE

n	α_{21}	$q_{1,n}/q_{1,1}$	$q_{2,n}/q_{2,1}$
1	0·729	1·000	1·000
3	0·797	0·985	1·080
7	0·906	0·964	1·201
15	0·955	0·954	1·253
31	0·978	0·950	1·278
51	0·987	0·948	1·287
101	0·993	0·947	1·293
∞	1·000	0·945	1·300

TABLE II

SHEAR FLOW RATIOS FOR THE THREE-CELL EXAMPLE

n	α_{21}	α_{31}	$q_{1,n}/q_{1,1}$	$q_{2,n}/q_{2,1}$	$q_{3,n}/q_{3,1}$
1	1·163	0·980	1·000	1·000	1·000
3	1·118	0·957	1·022	0·982	0·998
7	1·049	0·981	1·045	0·942	1·046
15	1·023	0·991	1·053	0·927	1·065
31	1·011	0·996	1·057	0·919	1·074
51	1·007	0·997	1·059	0·917	1·077
101	1·003	0·999	1·060	0·914	1·081
∞	1·000	1·000	1·061	0·912	1·083

of symmetry, the analysis is far from simple. It is therefore usual to further restrict the investigations to specific cross sections chosen from among those in current practice.

Beams having channel sections and subjected to bending moments and shear forces are analysed in this section. It is assumed that the material of the beams is governed by the non-linear stress-strain law of equation (6).

4.1. UNSYMMETRIC BENDING OF CHANNEL SECTIONS

The stress analysis of a channel section beam may be conveniently begun by a consideration of the section under the action of a bending moment. As shown in Fig. 7, the section is idealised by its median line. The moment vector M acts in the plane of the cross section and at a given angle β to the x axis. It is assumed that the Bernoulli-Navier assumptions remain valid, namely, plane sections normal to the undeformed longitudinal axis remain plane and normal to the deformed longitudinal axis. Since the material of the beam is assumed to follow a nonlinear stress-strain law (6), the neutral axis of the cross section does not pass through the centroid of the section as opposed to the case of the classical beam theory. The first step in the analysis is therefore to determine the location of this neutral axis.

Let the neutral axis make an angle α_0 with the axis of symmetry x. The point of intersection of these two axes is then chosen to be the origin 0 of both the co-ordinate systems x, y and x', y'. The sign convention of an angular co-ordinate is assumed positive in the counter-clockwise direction from the axis of symmetry x. The x', y' co-ordinate system may be obtained by rotating the x, y co-ordinate system through an angle α_0. The determination of α_0 and x_0 (Fig. 7) as functions of the given quantities β and M locates the neutral axis x'.

On the basis of the elementary beam theory assumption, the relation between the strain ε and the curvature κ is

$$\varepsilon = \kappa y'. \qquad (22)$$

This equation may be combined with the stress-strain relation (6) to yield the bending stress

$$\sigma = \lambda (\kappa y')^{1/n}. \qquad (23)$$

Since the beam is subjected to a pure bending moment, the stress distribution σ must give rise to a resultant moment M (or its components $M_{x'}$ and $M_{y'}$) and have a zero resultant force. These equilibrium conditions may be expressed with the use of equation (23) in the form

$$M_{x'} = \int_A \sigma y' dA = \lambda \kappa^{1/n} \int_A y'^{1+1/n} dA \qquad (24)$$

$$-M_{y'} = \int_A \sigma x' dA = \lambda \kappa^{1/n} \int_A x' y'^{1/n} dA \qquad (25)$$

$$\sigma = M_{x'} y'^{1/n} \Big/ \int_A y'^{1+1/n} dA = M_{x'} y'^{1/n} \Big/ I_{x',n} \qquad (26)$$

$$\int_A \sigma dA = \lambda \kappa^{1/n} \int_A y'^{1/n} dA = 0 \text{ i.e. } \int_A y'^{1/n} dA = 0 \qquad (27)$$

Figure 7 shows that $-M_{y'}/M_{x'} = \tan(\alpha_0 - \beta)$. Consequently, equations (24) and (25) may be combined to yield the relation

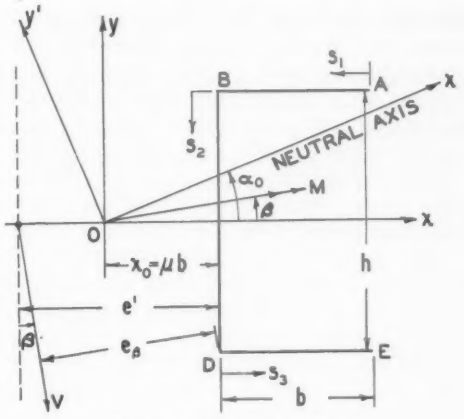

FIGURE 7. Channel section.

$$\tan(\alpha_0 - \beta) = \frac{\int_A x' y'^{1/n} dA}{\int_A y'^{1+1/n} dA} = -\frac{M_{y'}}{M_{x'}} \qquad (28)$$

The coupled nonlinear equations (27) and (28) may be used to determine α_0 and x_0 and thus, the neutral axis x'. This equation will now be applied to the channel section (Fig. 7).

From Fig. 7 the relation between the initial co-ordinates x, y and the rotated co-ordinates x', y' is seen to be

$$\left.\begin{aligned} x' &= x \cos\alpha_0 + y \sin\alpha_0, \\ y' &= -x \sin\alpha_0 + y \cos\alpha_0. \end{aligned}\right\} \qquad (29)$$

With the relation (29), the integrals in equation (27) and (28) may be evaluated. For instance, if the channel thickness t is uniform, then with the abbreviations $\mu = (x_0/b)$ and $\xi = h/2b$, equations (27) and (28) reduce to simultaneous nonlinear equations in α_0 and μ (*see* Ref. 11). The solution of these equations for a general n is prohibitively difficult. Therefore, solutions for only two specific values $n=1$ and $n=3$ are considered. For $n=1$, the solution for a general ξ is comparatively easy and leads to the well known result of the elementary bending theory. For $n=3$, since the solution of the equations is more involved, it is necessary to carry out the solution for particular values of ξ. As an example, $\xi=1$ has been considered. The results of the solutions

TABLE III

VARIATION OF NEUTRAL AXIS WITH MOMENT VECTOR FOR CHANNEL SECTION ($\xi=1$)

	$n=1$		$n=3$		
Direction of Neutral Axis α_0 *degrees*	*Direction of Moment Vector* β	$-\mu\tan\alpha_0$	*Direction of Moment Vector* β	$-\mu\tan\alpha_0$	e'/b
0	0°	0	0°	0	0·3500
15	2°25′	0·067	0°58′	0·034	0·3495
30	5° 8′	0·144	2°10′	0·072	0·3478
45	8°53′	0·250	4° 4′	0·134	0·3420
60	15° 8′	0·433	12° 5′	0·323	0·2864
75	30°17′	0·933	30°45′	0·670	0·2105
90	90°	—	90°	—	—

are presented in Table III. The parameter $\mu \tan \alpha_0$ shown is a measure of the vertical distance from the axis of symmetry to the point of intersection of the neutral axis and the web. The negative value of this parameter implies that the zero stress point is below the symmetric axis and the origin is to the right of the point C (Fig. 7). The variation of β with α_0 is also given in Table III for $n=1$ and $n=3$. The table shows that for small values of β there is a considerable difference in the location of the neutral axis obtained by the linear ($n=1$) and the nonlinear ($n=3$) analysis. For values of β approximately in the range 20° to 90° this difference is quite small.

Having determined α_0 and μ, *i.e.* the neutral axis for a given direction β of the moment vector M, equations (26) and (29) may be used to obtain the bending stress distribution

$$\sigma = \frac{M_{x'} y'^{1/n}}{I_{x',n}} = \frac{M\,[y\cos\alpha_0 - x\sin\alpha_0]^{1/n}\cos(\alpha_0-\beta)}{\int_A (y\cos\alpha_0 - x\sin\alpha_0)^{1+1/n} dA} \quad . \quad . \quad . \quad (30)$$

4.2. SYMMETRIC BENDING OF CHANNEL SECTIONS

In the preceding analysis of the channel section, the moment vector M was assumed to make an arbitrary angle β with the axis of symmetry. The resulting equations were difficult to solve for general values of n. However, for the particular cases when the vector M is parallel to the axis of symmetry ($M=M_x$, $M_y=0$, $\beta=0°$) or perpendicular to the axis of symmetry ($M=M_y$, $M_x=0$, $\beta=90°$), the analyses are considerably simpler.

In the first case when $\beta=0$, it is apparent that the applied moment vector, the neutral axis and the axis of symmetry all coincide. Thus, $\alpha_0=0$ and equations (27) and (28) can be shown to be identically satisfied. It follows from equation (30) that

$$\sigma = M_x y^{1/n} \Big/ \int_A y^{1+1/n} dA = M_x y^{1/n} \Big/ I_{x,n}. \quad . \quad (31)$$

The value of $I_{x,n}$ for the channel section (Fig. 7) is

$$I_{x,n} = 2bt\,(h/2)^{1+1/n}\,[1+\xi/(2+1/n)]. \quad . \quad (32)$$

When the moment vector is applied perpendicular to the axis of symmetry, *i.e.* $\beta=90°$, it can be shown that equation (28) is satisfied for $\alpha_0=90°$ and equation (27) assumes the relatively simple form

$$\mu^{1+1/n} - (1+\mu)^{1+1/n} - (1+1/n)\,\xi\mu^{1/n} = 0. \quad (33)$$

This equation determines $\mu = x_0/b$ for given values of ξ and n. For the particular value of $\xi=1$, the corresponding values of n and μ are shown in Table IV.

Finally, the stress distribution is (equation (30)) obtained as

$$\sigma = -M_y x^{1/n} \Big/ \int_A x^{1+1/n} dA = -M_y x^{1/n} \Big/ I_{y,n}, \quad (34)$$

TABLE IV

NEUTRAL AXIS FOR CHANNEL SECTION

n	1	3	7	15	31	51	101	∞
$-\mu$	0·250	0·155	0·095	0·054	0·029	0·018	0·010	0
$(e'/b)_{\substack{\beta=0\\ \alpha_0=0}}$	0·375	0·350	0·341	0·337	0·335	0·334	0·334	0·333

and for a channel section

$$I_{y,n} = 2tb^{2+1/n}\,\{\xi\mu^{1+1/n} + [(1+\mu)^{2+1/n} - \mu^{2+1/n}]\Big/(2+1/n)\}. \quad . \quad (35)$$

It may be noted that $I_{x,n}$ and $I_{y,n}$ as given by equations (32) and (35) reduce to the moments of inertia when $n=1$.

4.3. SHEAR STRESS DISTRIBUTION

The analysis has thus far been restricted to the investigation of the bending stresses. Now attention is turned to the determination of the shear stress distribution τ. To this end, the force equilibrium in the z-direction perpendicular to the plane section on which the shear force acts shows that

$$\partial\sigma/\partial z + \partial\tau/\partial s = 0,$$

$$\text{or} \qquad \tau = -\int (d\sigma/dz)\,ds. \quad . \quad . \quad (36)$$

Therefore, it follows from equation (26) that

$$\tau = -(1/I_{x'n}) \int (dM_{x'}/dz)\; y'^{1/n} ds. \quad . \quad (37)$$

With reference to Fig. 7, the shear force V and the resulting moment vector M act at angle β to the y and x axes, respectively. Consequently $\partial M_{x'}/\partial z = V_{y'} = V\cos(\alpha_0-\beta)$, and equation (37) finally yields the stress distribution

$$\tau = -\frac{V_{y'} Q_{x',n}}{I_{x',n} t} = -\frac{V\cos(\alpha_0-\beta)\int_0^s (y\cos\alpha_0 - x\sin\alpha_0)^{1/n} t\,ds}{t\int_A (y\cos\alpha_0 - x\sin\alpha_0)^{1+1/n} dA} \quad . \quad . \quad . \quad (38)$$

With the shear stresses determined, the location of the shear centre is investigated as the next step. By definition, the shear centre is the point through which the shear force V must act so that the resulting shear stresses give rise to a zero net twisting moment. It is well known that in the linear elastic theory the shear centre is a purely geometric property of the section, *i.e.* independent of the direction of the force V. Whether this is also the case in the nonlinear elastic theory requires verification.

With the geometrical quantities illustrated in Fig. 7, the condition of zero net twist moment may be written in the form

$$h\int_A^B \tau t\,ds + V e_\beta = 0. \quad . \quad . \quad (39)$$

Here, since the twisting moments are taken about the point D, it is apparent that the shear stresses in the lower flange and the web make zero contributions. Equations (38) and (39) may now be combined and simplified to yield

$$e_\beta/b = (2\xi V_{y'}/V I_{x',n}) \int_0^b Q_{x',n}\,ds. \quad . \quad (40)$$

Further, it can be shown e_β and e', the distance on the x axis of the point of intersection of the axis and the shear force V, have the trigonometric relation

$$e_\beta/b = \cos\beta\,(e'/b - \xi\tan\beta), \quad . \quad (41)$$

Finally, elimination of e_β/b between equations (40) and (41) and the use of expressions for $V_{y'}$, $Q_{x',n}$ and $I_{x',n}$ in equation (38) lead to

$$e'/b=$$

$$\xi\left\{\tan\beta+2\frac{\cos(\alpha_0-\beta)}{\cos\beta}\cdot\frac{\int_0^b\int_0^s(y\cos\alpha_0-x\sin\alpha_0)^{1/n}t\,ds\,ds}{\int_A(y\cos\alpha_0-x\sin\alpha_0)^{1+1/n}dA}\right\}$$

$$=\xi\left\{\tan\beta+2\frac{\cos(\alpha_0-\beta)}{\cos\beta}\cdot\times\right.$$

$$\left.\times\frac{\cos\alpha_0\int_0^b\int_0^s[h/2-(\mu b+b-s)\tan\alpha_0]^{1/n}t\,ds\,ds}{\int_A(y\cos\alpha_0-x\sin\alpha_0)^{1+1/n}dA}\right\}. \quad (42)$$

If $n=1$, the integrals in equation (42) may be readily evaluated in closed form. Such a procedure together with equation (28) leads to the simple result

$$(e'/b)_{n=1}=3/2\,(3+\xi). \quad (43)$$

It is apparent from equation (42) that e' (Fig. 7) is not only a geometric property of the section but also a function of the direction β of the shear force V. In other words, if the channel section of a nonlinear elastic material is to have a zero resultant twisting moment, the distance from the centre of the web to the point of intersection of the shear force and the axis of symmetry varies with the direction of the shear force. On the other hand, if the section is of a linear elastic material, equation (43) shows that this distance is independent of the direction of the shear force and locates the shear centre for a given geometry of the cross section. For $n>1$, the integrals in equation (42) may be evaluated numerically for given values of ξ and β (the corresponding values of α_0 and μ being evaluated from equations (27) and (28)). As a typical illustration, the variations of e'/b with β for $n=3$ and $\xi=1$ are shown in Table III.

When n has any value other than unity, the preceding analysis is considerably simplified for $\beta=0°$, *i.e.* for the shear force acting perpendicular to the line of symmetry. As a first step then, it follows from equation (38) that the shear stress for this particular case is given by

$$\tau=-\frac{V_y Q_{x,n}}{I_{x,n}t}=-\frac{V_y\int_s y^{1/n}t\,ds}{t\int_A y^{1+1/n}dA}. \quad (44)$$

Similarly, the distance at which the shear force V_y cuts the axis of symmetry is given by equation (42) as

$$(e'/b)_{\substack{\beta=0\\ \alpha_0=0}}=\frac{(2+1/n)}{2\,[(2+1/n)+\xi]}. \quad (45)$$

It may be trivially verified that the substitution of $n=1$ in equation (45) regains equation (43). Equation (45) gives the variation of e'/b with n for given values of $\xi=h/2b$. As an illustration, this variation is shown in Table IV for $\xi=1$.

While the preceding analysis has been concerned specifically with a channel section beam of a nonlinear elastic material, the extension of the method to other types of cross sections would not, as a rule, present any difficulty.

5. Thin-Walled Closed Sections Subject to a Shear Force

The discussion in Section 4 leads to the conclusion that the concept of the shear centre, as established in the elementary theory of structures has to be modified for open section beams of a nonlinear elastic material. In order to have a zero resultant twisting moment on a given open section, a shear force of arbitrary direction could no longer be applied at one fixed point. Moreover, for a given direction of application, the shear force distance from a fixed co-ordinate varies with the index n if the resultant twisting moment is required to be zero. In view of this, it is reasonable to investigate if a similar situation exists for closed section beams whose material is nonlinearly elastic.

The median line of the closed cross section of a beam subject to the shear force V is shown in Fig. 8, where t, in general a function of s, is the wall thickness at a generic point. The introduction of a fictitious cut transforms the section to an open one. Now, the shear flow $q=\tau t$ is a function of s. If the cut is made at $s=0$ and q_0 denotes the shear flow at this point, then at any point s the shear flow may be formally written

$$q(s)=q_0+q_1(s). \quad (46)$$

Here, q_1 is the shear flow in the open section and equals zero at $s=0$. While q_1 may be determined by the analysis of the preceding section, the generalised Castigliano's theorem is used to evaluate the indeterminate constant q_0. The first step towards this end is the determination of the complimentary energy per unit length of the beam.

The complimentary energy per unit volume of a material obeying the stress-strain law (6) is

$$\overline{U}_c=\int\varepsilon_{ij}d\sigma_{ij}=\int\varepsilon_{ij}ds_{ij}=CJ_2^{m+1}/(m+1).$$
$$=2CJ_2^{(n+1)/2}/(n+1). \quad (47)$$

In the presence of a shear stress τ only, $J_2=\tau^2=(q/t)^2$. Therefore, from equation (47) the complimentary energy per unit length of the beam is

$$U_c=[2C/(n+1)]\oint(q^{n+1}/t^n)\,ds. \quad (48)$$

The application of Castigliano's theorem to the energy expression in equation (48) yields

$$\partial U_c/\partial q_0=(\partial U_c/\partial q)(\partial q/\partial q_0)$$
$$=2C\oint(q/t)^n(\partial q/\partial q_0)\,ds=0. \quad (49)$$

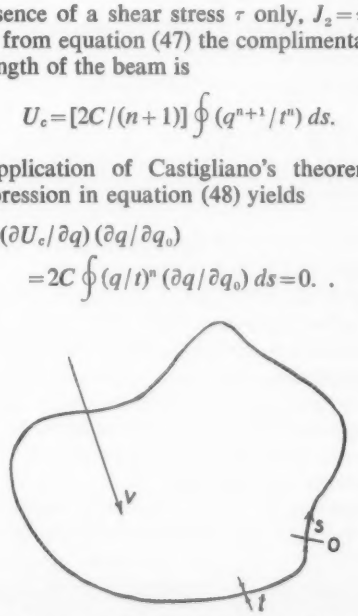

FIGURE 8. Thin-walled closed section.

Equation (46) shows that $\partial q/\partial q_0 = 1$, so that equation (49) reduces to

$$\oint (q/t)^n ds = 0, \quad . \quad . \quad . \quad (50)$$

and for a constant thickness

$$\oint (q_0 + q_1)^n ds = 0. \quad . \quad . \quad (51)$$

For integral values of n, the use of a binomial expansion reduces equation (51) to

$$\sum_{k=0}^{n} \frac{n!\, q_0^{n-k}}{k!\,(n-k)!} \oint q_1^k ds = 0. \quad . \quad . \quad (52)$$

q_0 is determined by the solution of equation (52) which is nonlinear for $n > 1$. Since q_1 can be obtained from the open section analysis (Section 4.3), the sum of q_0 and q_1, is the shear flow distribution by equation (46).

Once the shear flow distribution is determined, the distance from a fixed point at which the shear force must act to produce zero resultant twisting moment on the section can be obtained. As for the open cross section, this requirement is mathematically given by an equation of the type (39) which must be solved for the required distance for any given value of n. The method of analysis outlined will be illustrated by the solution of a particular example.

The example considered is the symmetric idealised aerofoil section shown in Fig. 9. The cross section is assumed to have a constant wall thickness with the curved portion being a semi-circle and the angle $\alpha = 15°$. The fictitious cut is introduced at $\phi = 0$ where the shear flow is q_0.

Since it is convenient to solve equation (52) in a dimensionless form, equation (46) is first written in the form

$$\eta = \frac{I_n}{4V_y t a^{1+1/n}} q = \frac{I_n}{4V_y t a^{1+1/n}} (q_0 + q_1) = \eta_0 + \eta_1. \quad . \quad (53)$$

Equation (44) may be used to determine η_1, so that

$$\left.\begin{aligned} 4\eta_1 &= \int_0^{\phi} \sin^{1/n}\phi \, d\phi, \qquad 0 \le \phi \le \pi/2, \\ 4\eta_1 &= \frac{\sqrt{\pi}}{2} \frac{\Gamma(0{\cdot}5 + 1/2n)}{\Gamma(1 + 1/2n)} + \frac{1 - [1 - (s_1/a)\sin\alpha]^{1+1/n}}{(1+1/n)\sin\alpha}, \\ & \qquad 0 \le s_2 \le a/\sin\alpha. \end{aligned}\right\} \quad (54)$$

Since the solution of equation (52) for a general n is not possible, particular values $n = 1$, 3, and 7 are considered. For these values, η_1 is evaluated from equation (54) and substituted into (52). This leads to a single algebraic equation in η_0 of order n in each case. Each of these equations has only one real root and these roots are, respectively, $-0{\cdot}433$, $-0{\cdot}523$, and $-0{\cdot}431$. Finally, equation (53) trivially determines the shear flow distribution η as the sum of η_0 and η_1.

The location of the shear force for a zero resultant twisting moment is the next step to be considered. With reference to Fig. 9, this requirement leads to the relation

$$2\left[\int_0^{\pi\alpha/2} q\, ds_1 + \int_0^{1/\sin\alpha} q \cos\alpha \, ds_2\right] = V_y(\delta/a - 1), \quad . \quad . \quad . \quad (55)$$

FIGURE 9. Idealised aerofoil section.

where the moment is taken about the convenient point 0.

Since equations (52) through (54) determine the shear flows q for given values of n, equation (55) may be readily solved for δ/a. For the three values of $n = 1$, 3, and 7, the corresponding values of δ/a are shown in Table V.

TABLE V

AEROFOIL SECTION—n AGAINST δa

n	1	3	7
δ/a	0·869	1·210	2·361

It is clear that δ/a, the distance of the application of the shear force V_y from a fixed point, shifts considerably if the requirement of a zero resultant twisting moment is to be satisfied.

6. Bending and Shear Analysis of Circular Cylinders

Although shear stresses arise, in general, as a natural sequence to the bending of a beam, their distribution on closed boundaries was discussed in the preceding section for the simple reason that the analysis of bending stresses on such boundaries is essentially the same as that presented for the open section. Therefore, with some differences in details, the results obtained in Section 4 could be readily applied to closed sections. As an illustration, the simple example of a beam with a circular thin-wall cross section is useful. In a practical sense, this cross section is the usual idealised representation of some aircraft structural components. Moreover, in a study of the effects of reinforcing the walls, a circular section is mathematically more tractable. With these considerations in view, unstiffened and stiffened circular cylinders are briefly discussed for bending and shear in the present section.

First, an unstiffened cylinder of radius R and uniform thickness t is considered. In view of polar symmetry of the cross section, the bending moment vector and the neutral axis coincide and pass through the centroid of the section. Consequently, the bending stress is formally given by equation (26). With the co-ordinates as shown in Fig. 10, $y = R\cos\phi$ and $dA = Rt\, d\phi$. Thus,

$$I_n = 4tR^{2+1/n} \int_0^{\pi/2} \cos^{1+1/n}\phi \, d\phi = 2\sqrt{\pi} tR^{2+1/n} \frac{\Gamma(1+1/2n)}{\Gamma(3/2+1/2n)}. \quad . \quad (56)$$

may be substituted into equation (26) and simplified to yield the dimensionless bending stress

$$\bar{\sigma} = \sigma t R^2/M = \cos^{1/n}\phi/(I_n/tR^{2+1/n}) = \frac{\Gamma(3/2+1/2n)}{2\sqrt{\pi}\,\Gamma(1+1/2n)}\cos^{1/n}\phi. \quad . \quad (57)$$

The symbol Γ in these equations represents the gamma function.

The shear force V is perpendicular to the neutral axis and is assumed to pass through the centre of the circle (centroid of the section). Before proceeding to obtain the shear stresses, it is relevant to point out that the recourse of introducing a fictitious cut (Section 5) need not be followed here since the problem considered has polar symmetry. Therefore, the shear stress distribution τ may be determined in a manner similar to that of the open section (see equation (38)). A simple consideration shows that

$$\tau = -VQ_n/2tI_n = -(VR^{1+1/n}/2I_n)\int_0^{\phi}\cos^{1/n}\phi\,d\phi. \quad . \quad . \quad . \quad (58)$$

Thus, the use of equation (56) leads to the dimensionless shear stress

$$\bar{\tau} = -\tau t R/V = \frac{\Gamma(3/2+1/2n)}{2\sqrt{\pi}\,\Gamma(1+1/2n)}\int_0^{\phi}\cos^{1/n}\phi\,d\phi. \quad . \quad (59)$$

The integral $\int_0^{\phi}\cos^{1/n}\phi\,d\phi$ can not be evaluated in closed form for a general value of n. Therefore, for given integral values of n, a numerical procedure is adopted to compute $\bar{\tau}$ in the range $0 \leq \phi \leq \pi/2$ of the upper limit.

The investigation is now extended to find the effect of reinforcing the cylinder considered (Fig. 10). The stiffeners are assumed to be of the same material as that of the cylinder and located 30° apart as idealised concentrated areas. An obvious consequence of the stiffeners is to increase the value of I_n as given by equation (56). This modified value of I_n may therefore

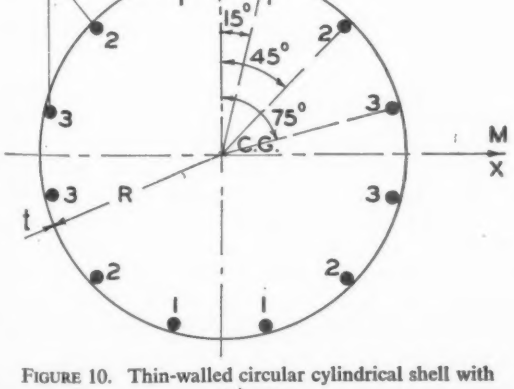

FIGURE 10. Thin-walled circular cylindrical shell with stringers.

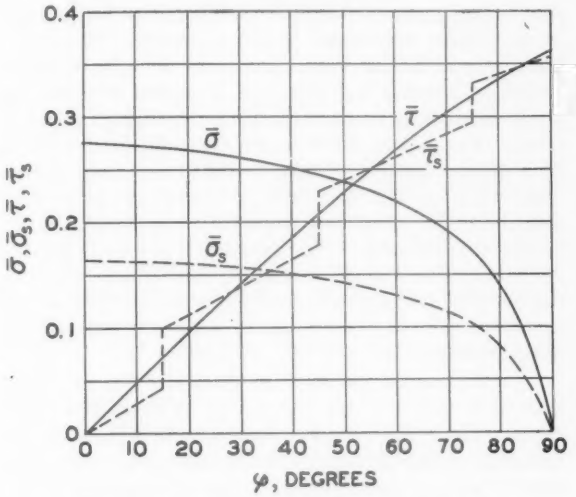

FIGURE 11. Stresses in circular cylinder ($n=3$).

be substituted into equation (57) to obtain the new dimensionless bending stress $\bar{\sigma}_s = \sigma_s tR^2/M$. Such a procedure yields

$$\bar{\sigma}_s = \frac{\cos^{1/n}\phi}{\dfrac{2\sqrt{\pi}\,\Gamma(1+1/2n)}{\Gamma(3/2+1/2n)} + 4(A_s/Rt)\left[0{\cdot}966^{1+1/n} + 0{\cdot}707^{1+1/n} + 0{\cdot}259^{1+1/n}\right]} \quad . \quad . \quad . \quad (60)$$

where A_s is the stringer area.

When a shear force V acts on the stiffened cylinder, the shear stress experiences sudden jumps at the stiffeners. With this modification, it can be shown that the non-dimensional shear stress $\bar{\tau}_s = -\tau_s tR/V$ follows from equation (59) as

$$\bar{\tau}_s = \frac{\int_0^{\phi}\cos^{1/n}\phi\,d\phi + (A_s/Rt)\sum_{i=1}^{N}\cos^{1/n}\phi_i}{\dfrac{2\sqrt{\pi}\,\Gamma(1+1/2n)}{\Gamma(3/2+1/2n)} + 4(A_s/Rt)\left[0{\cdot}966^{1+1/n} + 0{\cdot}707^{1+1/n} + 0{\cdot}259^{1+1/n}\right]} \quad . \quad . \quad . \quad (61)$$

Here, ϕ_i represents the location of those stiffeners that lie above a horizontal line passing through a particular ϕ. For instance, if $\phi = \pi/4 - \varepsilon$, where ε is an arbitrarily small positive number, only the two stringers marked 1 in Fig. 10 are included in the calculation of Q_n, so that $\phi_i = \pm 15°$. On the other hand, when $\phi = \pi/4 + \varepsilon$, the two pairs of stiffeners marked 1 and 2 must be used in the calculations and consequently $\phi_i = \pm 15°$ and $\pm 45°$.

Figure 11 shows the $\bar{\sigma}$, $\bar{\tau}$, $\bar{\sigma}_s$ and $\bar{\tau}_s$ distributions for the particular value $n=3$. In the case of the stiffened cylinder it was assumed that the total area of the twelve stiffeners was two-thirds the cross sectional area of the sheet. As might have been expected, the stress distributions for the two cylinders differ considerably.

7. Conclusion

In presenting the creep-stress distributions in thin-walled structures, the line of approach adopted has been

that these structures could be considered as being made of analogous non-linear elastic materials. The stress distributions in the latter structures have been determined by existing techniques of structural analysis and apply identically to corresponding structures subject to creep. The elastic strains must, however, be multiplied by a known function of time to obtain the creep strains. The value of the temperature-dependent time index determines the particular phase of creep for which the results are applicable. For instance, if $p=1$ (*see* Section 2), the constants λ and n assume the values Ω and k, respectively, and the solutions apply to the problem of steady creep. All other values of p apply to the primary creep problem.

The significant variation of the shear force distance with its direction of application for zero resultant twisting moment has already been noted. Further, the variation of this distance with the index n (or k) implies that if zero resultant twisting moment is ensured at a particular temperature, the cross section will experience a twisting moment with a change in temperature. Evidently, this twisting moment will give rise to additional shear stresses.

The main purpose of the paper has been the development of methods for the creep-stress analysis of thin-walled structures. Consequently, the examples have been limited to a few typical illustrations. The extension of the methods presented to other shapes of thin-walled structures would not, as a rule, present too much difficulty.

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Transonic Buffeting on Aerofoils*

by

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Introduction

There are many problems to be solved when it is required to fly an aircraft at speeds approaching the speed of sound. When the speed of the aeroplane exceeds its critical Mach number, regions of supersonic flow appear at points on its surface, where the velocity of the displaced air is greatest. These regions of supersonic flow increase in size with increasing speed until the flow is wholly supersonic. The range of speed in which this mixed flow occurs is called the transonic range. In this range an aircraft will experience a sudden increase in drag, trim changes, loss of control effectiveness and possibly instability, or buffeting.

The purpose of this paper is to discuss only the last of these problems and, to simplify the discussion still further, it is proposed to neglect wing root and tip effects and to consider buffeting only in relation to aerofoils of infinite span so that the flow is two-dimensional. While avoiding mathematical detail, we shall look at the cause of buffet, its theoretical prediction and its effect on design methods.

NOTATION

P	pressure
H_0	stagnation pressure (total head)
U	stream velocity
u	local velocity within boundary layer
δ^*	displacement thickness $= \int_0^\infty \left(1 - \frac{u}{U}\right) dy$
θ	momentum thickness $= \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$
δ^*/θ	form factor

Suffixes

0	free stream conditions
1	conditions immediately upstream of the shock wave
TE	trailing edge conditions

What is Buffet?

Buffet is the term used to describe the aerodynamic shaking normally experienced by an aircraft either during high speed flight or near the low speed stall. Transonic buffeting was known as early as 1938 when it was referred to as " Compressibility Burble " and high performance wartime aircraft like the Spitfire sometimes experienced it in high speed dives. The immediate cause of this phenomenon is the rapid and irregular pressure variation in the turbulent " dead air " region which is found downstream of the point where the boundary layer separates from a solid surface.

Buffeting is caused, therefore, by boundary layer separation, whether this occurs in a low speed stall or as a result of some transonic effect. We shall now examine the transonic flow regime more closely to discover what causes the boundary layer to separate.

Flow Description

In any moving stream of air which passes over an obstruction, there must be a point in the wake where the stream reunites after being divided by the passage of the object. This is often called the critical point. In the case of a conventional aerofoil, when the boundary layers have not been caused to separate permanently from the surface, this point is at the trailing edge, while in separated flow it is farther downstream.

At the critical point the flow must always satisfy two conditions. Firstly, there must be equality of pressure between the two streams and secondly, the difference between the local pressure and the free stream static pressure $(P - P_0)$ must be such that the momentum distribution in the shear or boundary layers combined in the wake is able to support this pressure change between the point considered and a point far downstream. This is known as the compatibility condition and has been expressed in terms of the usual boundary layer parameters as follows[2]:—

$$\frac{V_{TE}}{V_0} = \frac{P_1}{P_{TE}} \frac{\delta^*_1}{\delta^*_{TE}} \left(\frac{\delta^*}{\theta}\right)_{TE}.$$

The pressure equality condition is the well-known Kutta condition of potential theory, while the compatibility condition expresses the effects of viscosity.

In purely subsonic flow both conditions are satisfied by smooth continuous flow over both surfaces and stalling occurs when the compatibility condition can no longer be satisfied at the trailing edge. Supersonic flow satisfies both conditions at or near the trailing edge by adjustment of the trailing edge shock waves.

Transonic flow, however, is more complex and we need to examine it in more detail. The discussion is restricted to the upper surface, where the effects are usually greater.

With reference to Fig. 1 it is seen that after an initial rapid expansion from stagnation conditions at the leading edge the flow reaches sonic velocity. The expansion continues in a manner similar to Prandtl-Meyer flow, but is still affected slightly by downstream conditions due to the limited extent of the supersonic region normal to the surface. A more important effect (Fig. 2) is that of expansion waves from the front part of the surface, which are reflected at the upstream boundary of the supersonic region. Some of these return to the aerofoil surface behind the crest and are reflected

*Read before the Brough Branch in April 1959 and awarded the 1960 N. E. Rowe Medal for the 21-26 years of age group.

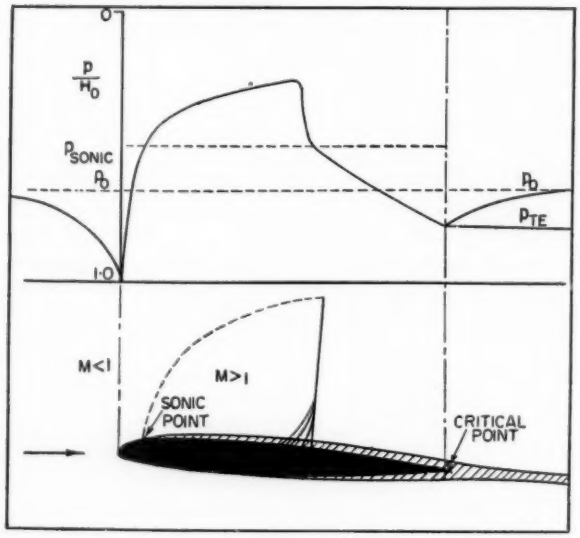

FIGURE 1. Transonic flow without separation.

again as compression waves, thus causing a considerable reduction in the overall expansion.

At some point (Fig. 1) a normal shock wave occurs, which is always found to raise the pressure to a value corresponding approximately to sonic velocity. This enables the air to undergo a subsonic compression over the remainder of the surface and to satisfy the trailing edge conditions.

As the speed increases at constant incidence, the shock wave moves toward the trailing edge and its strength increases with the upstream Mach number until the associated pressure rise so weakens the velocity profile of the boundary layer that the compatibility condition can no longer be satisfied at the trailing edge. When this occurs, the boundary layer separates under the foot of the shock (Fig. 3) and a " bubble " of dead air appears which spreads backwards past the trailing

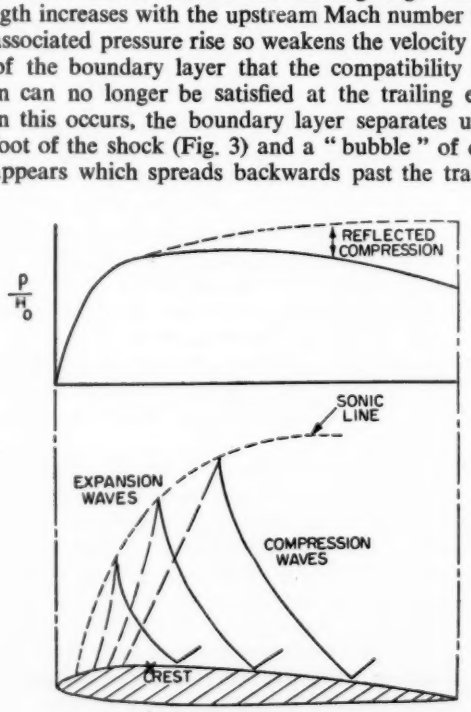

FIGURE 2. Reflected compression effect.

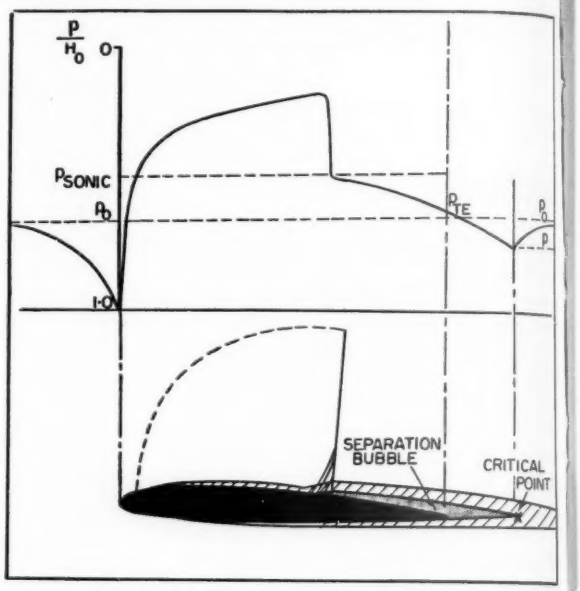

FIGURE 3. Transonic flow with separation.

edge and into the wake. The onset of buffeting is found to coincide with the arrival of the separated region at the trailing edge.

The consequent divergence of trailing edge pressure is often used to detect buffeting in wind tunnel tests. (Fig. 4.)

Buffet Prediction

Because of the complexity of " mixed " transonic flow, there is no exact theory which is suitable for the prediction of the onset of buffeting. Several empirical or semi-empirical methods have been devised, but none appear to be in general use. All of them depend upon some criterion to determine the shock strength which will cause a given boundary layer to separate. Experimental results have been quoted suggesting values of M, which vary between 1·23 and 1·27. Mager[4] used conventional boundary layer theory to predict that the value of M_1 to cause separation was 1·20. A further

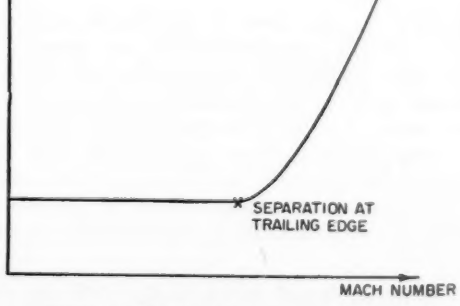

FIGURE 4. Trailing edge pressure divergence.

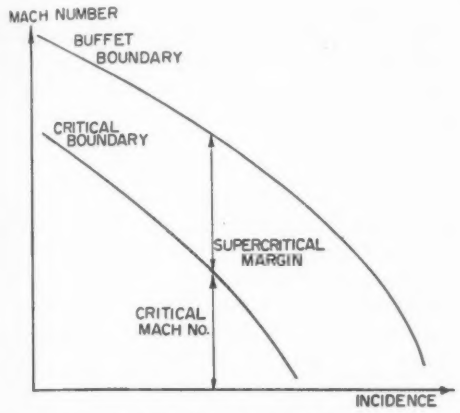

FIGURE 5. Critical Mach number and buffet boundaries.

advance has been made recently by relating the separation shock strength empirically to the shape of the leading edge of the aerofoil.

It is also found that, on many types of aerofoil, a "Mach number freeze" occurs so that the value of the stagnation pressure ratio P/H_0 does not vary appreciably with Mach number in the transonic range. In these cases we can calculate the pressure distribution due to a sonic stream by the method of Ref. 3 and use empirical methods to estimate the small differences between this and the pressure distribution upstream of the shock at the required Mach number. If this is used together with a measured or calculated subcritical distribution, corrected to the same Mach number and used to represent conditions downstream of the shock, then a suitable empirical shock relation giving downstream pressure in terms of upstream pressure can be used to find a unique shock position and strength for each case. It can then be decided whether or not the boundary layer will separate for a given Mach number and configuration.

Design Considerations

Obviously it is impossible in this short paper to give anything more than a brief account of the way in which the buffet problem influences aerofoil design.

To design an aerofoil to operate in the transonic

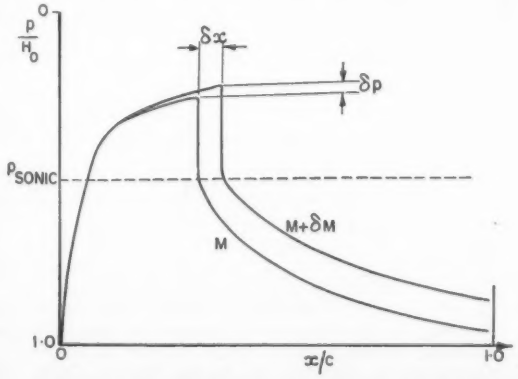

FIGURE 7. Effect of downstream pressure gradient.

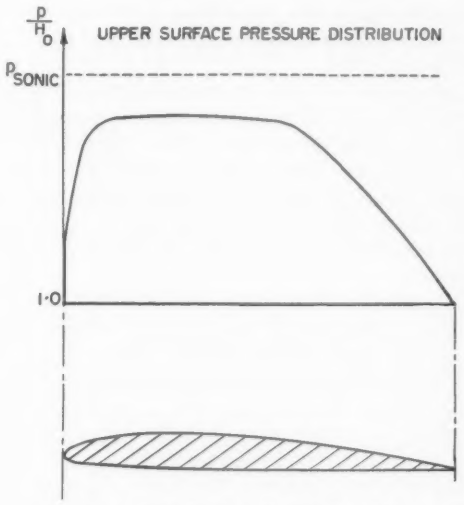

FIGURE 6. "Rooftop" section for high M_{crit}.

range, it is essential to prevent the onset of buffeting within the flight envelope regardless of what may happen when buffeting does occur. For supersonic flight, on the other hand, it is necessary either to eliminate buffeting throughout the transonic range or, failing this, to reduce its intensity to a tolerable level, assuming that the aircraft will only fly in this condition during short periods of acceleration or deceleration.

Considering the transonic case, it is possible to consider the buffet Mach number as the sum of two parts; the critical Mach number, and a supercritical Mach number increment (Fig. 5). Until quite recently it was thought to be sufficient to consider only the critical Mach number and to attempt to raise this by reducing the maximum supervelocities over the aerofoil surface in subcritical flow.

This was achieved by designing "rooftop" sections (Fig. 6) having flat-topped pressure distributions of considerable chordwise extent at the design lift coefficient. However, it has been found that this method tends to reduce the supercritical Mach number margin because sonic velocity is reached simultaneously over the whole

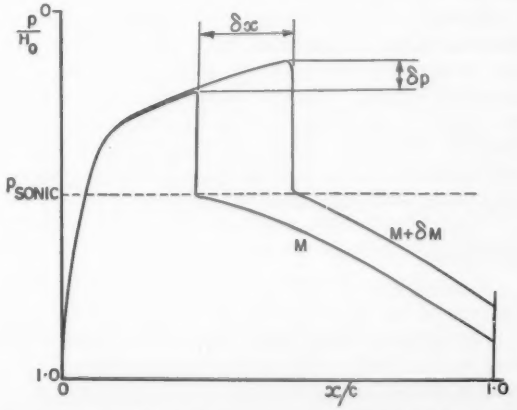

FIGURE 8. Effect of downstream pressure gradient.

flat-topped region and supersonic expansions are rapid. There may therefore be no improvement at all in the buffet boundary.

Considering only the supercritical margin it appears that the best aerofoil shape will have a minimum of curvature aft of the sonic point, thus reducing the supersonic expansion. This leads to a thin wing with a small trailing edge angle and it has been suggested[1] that, if the flow deflection at the trailing edge is less than $4\frac{1}{2}°$, no serious effects of separation will appear at any Mach number.

To make the best use of the reflected compression effect mentioned earlier, very careful attention must be paid to the shape of the leading edge. There is still much speculation and experimental investigation on this subject and no rules can be made. Current evidence indicates that a compromise must be sought or a choice made between two conflicting effects.

A thin, sharp leading edge gives a very sudden expansion at the front. Reflection of this produces a favourable reduction in the expansion and hence in the shock strength farther aft. On the other hand, a thick, well-rounded leading edge appears to have a beneficial effect on the boundary layer so that a stronger shock wave can be tolerated without separation.

This choice will be influenced by the fact that, if the aerofoil is designed to give a steep rise in the subcritical pressure distribution in the area where the shock wave occurs, its rearward movement with increasing Mach number will be reduced. This not only tends to prevent an increase in shock strength, but also keeps the shock in a forward position where the boundary layer is stronger.

Buffeting is not, of course, the only factor influencing aerofoil design. There are other transonic problems, and also the aircraft will have to fly slowly during take-off and landing. Due consideration must be given to these factors and this may lead to the adoption of methods other than section design, to prevent buffeting. These come under the heading of boundary layer control.

Vortex Generators

A row of small plates set normal to the aerofoil surface and inclined at a small angle to the stream direction sheds a series of vortices similar to the tip vortices from a finite wing. If the plates are of suitable size and are spaced at frequent intervals along the span in a position just ahead of where the first serious shock wave is expected to appear, then the energy added to the boundary layer by the action of the vortices renders it less likely to separate. Vortex generators have been used with considerable success in the past, though there is a drag penalty associated with their use.

Blowing

If a supply of high pressure air is available as in the case of an aircraft powered by gas turbine engines, it is possible to blow a jet of air backwards over the wing surface from a slit or from a row of holes. Energy is thus added to the boundary layer making it better able to withstand the pressure rise across a shock wave.

The position of the blowing slit or holes is important. The maximum effect for a given quantity of air at a given velocity is obtained when the blowing is applied just behind or under the foot of the shock waves[5].

Suction

A very similar effect can be produced if the boundary layer is removed through the skin. This enables a new boundary layer to form which of itself is able to withstand a stronger shock wave than the old one. This last method has some disadvantages, the most obvious being the relatively low pressure required to remove air from the aerofoil surface where the pressure is already lower than the free stream pressure.

Both suction and blowing do, of course, absorb some power from the engines. This is equivalent to the drag penalty incurred by the use of vortex generators. Because of this power loss, methods of boundary layer control should be regarded as being secondary and supplementary to good aerofoil design. They can always be used to give additional improvement on any aerofoil.

Conclusion

Our discussion has necessarily been restricted to the consideration of aerofoils in two dimensions, but it should be noted that the plan form and loading of a wing play a very important part. The correct use of sweepback and wing twist, together with carefully designed wing roots and tips, could give far more improvement than is possible by section design alone.

In this paper it has been stated that transonic buffeting on aerofoils is caused by shock-induced separation of the boundary layer. This separation occurs because, given certain combinations of aerofoil shape, speed and incidence, the flow is unable to satisfy the essential conditions of pressure equality and compatibility at the trailing edge. Instead, the conditions are satisfied downstream at a point marked by the junction of the shear layers from the two aerofoil surfaces. To design aerofoils which avoid this separation we need to be able to estimate when it will occur. In the absence of an exact theory, several empirical methods have been invented and we have looked at one promising method of approach. Finally, some of the possible design measures have been discussed, together with methods of boundary layer control used to delay or prevent separation.

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A History of the Development of Flight Refuelling*

by

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1. The Origins

Whenever a new and truly great idea is put forward for the first time it is usually received with scorn and derision by those whom it directly concerns. Such was the initial reception of the idea of refuelling aircraft in flight.

Soon after the First World War air carnivals became very popular around the flying fields of the United States of America and it was in a search for new stunts that two intrepid fliers hit on the idea of transferring fuel by hose pipe from one aircraft to another. The two single-seater aeroplanes flew one above the other, the upper one carrying the extra fuel; in order to transfer it the pilot threw a length of hose overboard leaving it trailing behind him. The receiver aircraft then manoeuvred into position and the pilot caught the hose and put the nozzle into his reserve fuel tank. When a small quantity of fuel had been transferred, he pulled out the hose and threw it clear of his aircraft, leaving the donor aircraft to haul it in.

2. Early Experiments

After several air shows, this stunt was seen by the Officer Commanding Rockwell Field, San Diego, California, Major (now General) H. H. Arnold, who immediately realised the far-reaching possibilities that flight refuelling, using a more positive and foolproof method of making contact, could have if applied to military and civil aviation. He initiated experiments at Rockwell Field with some two-seat, single-engined Army D.H. 4Bs. Two such aircraft were equipped as tankers, having a 50 ft. length of hose with a quick acting shut-off valve which enabled the pilot to establish contact while flying above the receiver aircraft. The receiver aircraft themselves had additional fuel tanks behind the rear cockpit, the tanks having large openings to facilitate fuel transfer.

3. A Record Duration Flight

After a long session of petty bickering in official channels, Major Arnold gained approval for his experiments and on the 20th April 1923 the first flight was made from Rockwell Field. A hose was successfully passed and the tanker and receiver flew together in contact for some 40 minutes, although no fuel was actually transferred. Throughout May and June of that year numerous trial flights were carried out, during which the equipment used was thoroughly tested and a large quantity of fuel was passed. The results of these trials were so satisfactory that on 27th June an attempt was made on the existing flight endurance record. The D.H. 4Bs were used and two contacts were made, 25 gallons of fuel being transferred at the first and 50 gallons at the second, but the flight had to be abandoned after 6 hours 29 minutes owing to the failure of an electrical generator. After another attempt on the record had failed for a similar reason two fliers stayed airborne for 37 hours and 15 minutes, thus establishing 14 world records. Soon after this, the first point-to-point refuelled flight was made from Canada to New Mexico, taking about 12 hours with two contacts being made.

It was later in 1923 that the only instance known to date of a fatal crash due to refuelling procedure occurred. During an air carnival at Kelly Field, the tanker's hose became entangled in the propeller of the receiver aircraft which subsequently crashed, killing the pilot outright.

4. More Records

The first aerial refuelling over Europe took place in December of 1923 at Le Bourget in France, but, after this, there was a complete lull in activities for nearly five years before the American War Department sponsored more duration flights. The beginning of a sudden spate of record-breaking flights came in January of 1929 when a famous aircraft, the Air Corps Fokker trimotor monoplane, Question Mark, with its crew of five, took off from Los Angeles and was supplied with fuel, oil, food and water by two Douglas biplanes. It remained in the air for 150 hours. This flight was followed by 14 more during the same year and by the end of the year the record was held by a Curtiss Robin monoplane and stood at 420 hours. Early the next year a Stinson Detroiter flew 41,500 miles in 553½ hours and this was quickly followed by a 647½ hour flight by Jackson and O'Brine in their monoplane Greater St. Louis.

5. The Atcherley Method of Making Contact

While these duration flights were being made, an Englishman was in the U.S.A. to take part in the

*Read before the Brough Branch in April 1959 and awarded the 1960 N. E. Rowe Medal for the under 21 years of age group.

National Air Races. He was Squadron Leader Atcherley of the R.A.F., and after he had witnessed some of these duration flights, he realised the great significance of being able to refuel an aircraft in flight. He foresaw great possibilities, but he also considered that the contacting system used was not the most satisfactory that could be devised, and so when he returned to his squadron in Transjordan he set to work devising a simpler and safer method of making contact. The results of his work were patented internationally, but he was refused permission by the Air Ministry to make an attempt on the world duration record using his new and simplified equipment.

In the Atcherley method, the aircraft to be refuelled trailed a 300 ft. long horizontal line to the end of which was fastened a three-pronged grapnel, while the tanker trailed a 100 ft. weighted line to an angle of about 70 degrees to the horizontal. The flying procedure was for the tanker to approach the receiver from above, behind, and to one side, and then to cross the receiver's flight path, thus causing the two lines to cross and the grapnel to make contact with the weight. The receiver then winched in the line and hauled a hose across from the tanker. Fuel was passed and the hose and weighted line were taken in again by the tanker, which was then free to turn for home. This system has been used on occasions, but has not been extensively adopted by either military or civil aviation.

6. Sir Alan Cobham and the Courier

During December 1931, an important contribution was made to the progress of flight refuelling in the form of a paper by J. H. Larrard of the Royal Aircraft Establishment, Farnborough, which appeared in the *Journal of the Royal Aeronautical Society* of that month In it were indicated the commercial possibilities of flight refuelling, showing how the range and payload of aircraft could be increased and the take-off performance improved.

Soon after this, Sir Alan Cobham began aerial refuelling experiments with two D.H. 9s. Ever since 1932, Sir Alan has continued the development of this branch of aviation enthusiastically and persistently and his company, Flight Refuelling Ltd., have been, and still are, pioneers in their field.

Further tests were carried out using a D.H. 9 and a Handley Page W.10 biplane converted into a tanker, and then preparations were made, in conjunction with the Air Ministry, for a non-stop, flight-refuelled journey from England to India. The aircraft was to be an Airspeed Courier, a single-engined, low-wing monoplane and for the first time the difference between maximum take-off weight and maximum flying weights was to be exploited by refuelling just after take-off. The take-off could be made with little fuel on board and then the tanks could be topped up to maximum capacity. The aircraft's maximum take-off weight was 3,700 lb. and the maximum flying weight 5,050 lb., so that an increase in weight of some 35 per cent was possible.

At 6.32 a.m. on the 22nd September 1933 the Courier set off piloted by Sir Alan Cobham and Squadron Leader Helmore and was immediately refuelled by a Handley Page W.10. The following day it was met by another W.10 over Malta, where another contact was successfully made, but unfortunately the engine throttle linkage broke soon afterwards and Cobham was obliged to make an almost powerless, wheels-up, forced landing at Hal Far in Malta. The system used on this flight was for the tanker to lower a toy balloon filled with half a gallon of water which was caught by the co-pilot of the Courier, who then hauled the fuel hose down by means of the line attached to the balloon. The co-pilot was also obliged to hold the hose nozzle into the reserve tank.

7. Another Flight Duration Record

Also in 1933, the brothers Al and Fred Key made their famous flight in which they stayed aloft for four weeks and captured the duration record which had stood since 1930. The Key brothers took off on 4th June in a Curtiss Robin from Meridan, Mississippi and landed again on 1st July, after flying continuously for 653½ hours. During the flight, 400 contacts were made for the transfer of fuel, oil, food and water.

8. The First Scheduled Flight-Refuelled Service, 1939

The first " refuelled in flight " scheduled airline service was organised with the co-operation of Imperial Airways. Trials were made over Southampton Water —some in very bad weather—using an Armstrong Whitworth AW 23, and a Short " C " class flying boat, the Cambria. The technique had again been modified, and now the " Ejector " Method was being used. The receiver trailed a weighted line and a line from the tanker was fired across the receiver line so that it became gripped in the pawls of a grapnel. The tanker line was then hauled back and replaced by the fuel hose which the receiver then had to haul back into itself. To break the contact the tanker hauled the hose and its own line back and then turned away from the receiver causing the line to break at a preselected weak link.

Eight special flying boats were built to operate the North Atlantic route, two of which, Caribou and Cabot, had flight refuelling equipment installed, and four Handley Page Harrow bombers were converted into tankers. The Royal Mail Boats took off from Southampton and then alighted at Shannon for about an hour before making the Atlantic crossing. The first refuelling took place after take-off at Shannon, giving the aircraft enough fuel to reach Botwood (Newfoundland) against the prevailing westerly winds. After a short stop at Botwood the flight went to Montreal and thence to New York. On the return flight the flying boats were refuelled after take-off at Botwood by tankers based at Gander.

Altogether 16 crossings were made in the course of these elaborate trials and, on all but one of them, successful contacts were made between the tanker and the receiver. On the remaining occasion, the prevailing tail

wind was so strong that the flying boat made the crossing without recourse to flight refuelling.

The great success of these trials had convinced Imperial Airways of the soundness of the system and they decided to add more aircraft to their fleet and to extend the North Atlantic service, but unfortunately the Second World War put an untimely end to these activities, and all the plans collapsed.

When the U.S.A. entered the war in 1941, the U.S.A.F. issued a requirement for a long-range bombing team and Flight Refuelling Ltd. were approached to adopt a B-17 Flying Fortress, and a B-24 Liberator to form such a team. The B-24 was made the tanker and the loop-hose installation was fitted. However, the U.S.A.F. did not adopt the system, although extensive flight testing had been carried out at Eglin Air Force Base, Florida.

9. The Tiger Force Project

Quite suddenly, during 1944, the same company was approached by the Air Ministry and ordered to modify 600 Lancasters as tankers and 600 Lincolns as receivers, to allow the Tiger Force to bomb Japan from bases in South East Asia but, just as suddenly, the Americans offered the R.A.F. an island within the Lincoln's unrefuelled striking range of Japan and the contract for the modifications was cancelled. Although it was not a practical demonstration of the great value of in-flight refuelling, the Tiger Force project was a considerable step towards official recognition of the system and, after the war, the Ministry of Civil Aviation was not slow to take advantage of the experience gained and of the equipment that had been produced for the Tiger Force.

10. English Channel Trials, 1946

Progress continued with more trials using British South American Airways (B.S.A.A.) Lancaster aircraft as receivers. Flight Refuelling Ltd. flew converted Lancasters as tankers from Ford Aerodrome in Sussex and the B.S.A.A. aircraft were intercepted at predetermined points for the transfer of about 1,000 Imperial gallons of fuel. The procedure adopted was for B.S.A.A. to telephone Ford aerodrome some two or three hours before the flight started, advising the tankers of the course to be taken by the receivers and indicating the height, time and position desired for the interception. Forty interceptions were made during the trials at heights varying from five to twenty thousand feet.

11. Bermuda Trials, 1947

The reliability of the method of bringing two aircraft together by radio and radar had been successfully demonstrated during the English Channel trials and it was then decided that final proof of the technique of flight refuelling should take the form of long distance, trans-ocean trials and the Ministry of Civil Aviation undertook to sponsor a series of flights from London to Bermuda. The success of a flight-refuelled airline service depends primarily on being able to make contact with the tanker at a pre-determined rendezvous and secondly on being able to receive the required amount of fuel in the shortest possible time without let or hindrance to the receiver aircraft.

Tanker bases were set up at strategic points on the route and a scheduled service, consisting of eleven weekly return flights, began on 28th May 1947 and concluded on 11th August. All the flights were successfully refuelled except one, when a Lancaster receiver landed at the Azores base with engine trouble.

The interception technique used was as follows. The receiver's flight plan was signalled to the tanker base before take-off after which position reports were sent from the receiver stating the E.T.A., fuel required and position of rendezvous. W/T communication between the two aircraft was established at a separation of about 300 miles and later V.H.F. communication when they were between 70 and 100 miles apart. Radar interception was made about the same time as V.H.F. contact.

These trials, and the subsequent ones over the North Atlantic route during 1948, proved conclusively the value and reliability of the flight refuelling system in all weather conditions. There was no occasion on which a tanker failed to take off and only one interception failed due to the failure of the cabin heating system in one of the tankers.

Thus there is little doubt that the degree of regularity of an airline service can be greatly increased by the adoption of the flight refuelling system.

12. The First Round the World Non-Stop Flight

During 1948, loop-hose equipment for flight refuelling was supplied to the United States Government for the conversion of a team of B-29s and B-50s to tanker and receiver aircraft and, in December of that year, a flight was made from Carswell Air Force Base, Fort Worth, Texas, to Hawaii and back, carrying a dummy bomb load. The total distance flown by the aircraft was about 9,400 miles. Soon after this, the 43rd Air Refuelling Squadron of the U.S.A.F. achieved the first air refuelled, non-stop, round the world flight with another B-50, Lucky Lady II.

The aircraft took off from Carswell A.F.B. on the 26th February 1949 and landed there again on 2nd March, after flying 23,108 miles in 94 hours one minute. It was refuelled over the Azores after 3,800 miles, Dhahran after 5,200 miles, the Philippines after 4,900 miles and Hawaii after 5,300 miles. At each base there were four tankers in readiness, two to make the actual transfer and two more in reserve.

The Boeing Airplane Company, who installed the loop-hose equipment for this flight, then set to work to devise an entirely new system of making contact called the " flying boom," which will be described later.

13. Two New Systems for Refuelling in Flight

Early in 1949 the United States Air Force issued a requirement for refuelling equipment to be used in conjunction with single-seat, jet-engined fighters. This

presented immediate problems since the receivers would have no crew to operate the conventional loop-hose equipment, so that the system used would have to demand no more from the receiver pilot than normal service flying skill and contact would have to be instantaneous.

13.1. PROBE AND DROGUE METHOD

An entirely new technique was devised by Flight Refuelling Ltd., taking only four months to develop, the first flying contact being made early in April 1949. This system is now operational with both the R.A.F. and the U.S.A.F. and is known as the probe and drogue method. The initial idea was so sound that a record duration flight for jet-propelled aircraft was made by a Gloster Meteor, using a Lancaster tanker, on 7th August 1949. During its twelve-hour flight the Meteor was refuelled ten times in steadily deteriorating weather, the last contact being made in cloud.

The equipment required by the receiver aircraft is almost negligible compared with that required for the loop-hose method, as it consists only of a length of fuel pipe at one end of which is a nozzle, the whole being encased in a streamline shroud. This forms the probe and it can be fitted to the aircraft at any convenient point where there is a tapping provided to connect up with the existing fuel line. With such a probe fitted, the aircraft is ready to receive fuel. The normal locations for the probe are in the nose of the aircraft, or the leading edge of a wing in the case of machines with nose air intakes, and in certain specialised cases, retractable probes are used.

The tanker's equipment consists of a drogue, a hose, and a drum. The drogue is a lightweight conical receptacle incorporating a set of spring-operated toggles which automatically grip the probe nozzle when the latter is flown into the drogue. The drogue is attached to the hose, which in turn is stored on a variable torque input power-driven drum, and the equipment functions as follows. When the operator in the tanker selects the unit for operation, the hose is allowed to trail and all subsequent operations are automatic. The receiver pilot, who need only have the skill to formate on another aircraft, looks for an amber light mounted on the base of the hose unit to indicate that all is ready and then flies the probe into the drogue, which, due to its symmetrical shape, is trailing steadily. The toggles engage and fuel flow is automatically started, replacing the amber light with a green one indicating that all is well. Two red warning lights are used to advise the receiver pilot of an emergency. When the fuel transfer is complete, the receiver pilot applies his air brakes, or just throttles back, and at a predetermined tensile load the probe disengages itself from the toggles and the fuel flow is automatically shut off.

In the tanker, all the operating is done by pressing a series of switches, mounted on a small panel, which control the trailing and rewinding of the hose, and the opening and closing of the main fuel storage tank cocks.

The length of the hose is between 75 and 100 feet and the receiver aircraft can maintain contact with the tanker within an envelope bounded by a 30° cone, the apex of which is at the point where the hose leaves the drum. A segment of this cone is lost because the receiver must stay out of the tanker's slipstream, and there is also a limit to the distance that can separate the aircraft since the receiver cannot close right up to the tanker. However, there is still a very high degree of freedom for relative movement between the two aircraft, which is invaluable in adverse or "bumpy" weather. The two main advantages of this equipment are :—

(*i*) its suitability for use by single-seat fighter aircraft,

(*ii*) its adaptability to different types of aircraft.

The tanker's equipment can readily be accommodated in the bomb bay of a Valiant or a Canberra, and bleeds can be taken off the aircraft's existing fuel storage tanks.

This second feature has a more far-sighted advantage than may at first be realised. When a particular type of aircraft, invariably multi-engined, is used as a universal tanker for many types of aircraft the air speed and altitude of any receiver are governed by the tanker's own performance. This factor would be prohibitive with modern aircraft since the development of potential receivers has far outstripped that of tankers, and a term known as tanker-receiver compatibility has arisen. An example of this was seen in the standard practice by the U.S.A.F., that when a KC-97 Stratotanker was refuelling a B-47 Stratojet the two aircraft had to go into a shallow dive in order to maintain contact. Many solutions were tried, such as fitting turbo-jets to KB-29s and KC-97s, but these aircraft were basically incompatible with the jet-propelled bombers and no compromise could be fully satisfactory. However, the ultimate solution to this problem is provided by the readily transferable hose reel unit, since it can be fitted easily to an aircraft of the same type as the receiver, so that relative performance does not enter into consideration.

This concept of pairs of similar aircraft operating as a tanker/receiver team cannot be applied to single-engined fighters and other types of small combat aircraft, because they cannot carry enough fuel, but the most modern types of bomber can be adapted as tankers for the fighters so that the difference in performance is a minimum.

13.2. PROBE AND DROGUE TRIALS WITH THE U.S.A.F.

The probe and drogue method was the result of a U.S.A.F. specification and the service adopted it enthusiastically for fighter aircraft. Many tests were carried out at Eglin A.F.B., Florida, using B-29s and B-50s, and soon after their institution a three-drogue version of the B-50 was undergoing flight trials. Hose reels were fitted, one in the bomb bay, the other two in fairings on each wing tip. Extra fuel storage tanks were fitted in the bomb bay and under the wings so that the aircraft could then carry enough fuel to service seven F-100s in one flight. The tests were carried out both by day and night, the night operations being facilitated by the fitting of several spot lights on the tanker.

one of which, set in a rotating mounting on the top of the rudder, could be seen for 50 miles on a clear night.

Although the U.S.A.F. and the U.S. Navy use probe and drogue extensively for refuelling fighters, their bombers use the Flying Boom System developed by the Boeing Airplane Company after the round the world flight by Lucky Lady II.

13.3. BOEING FLYING BOOM SYSTEM

The tanker's equipment for this system consists of a semi-rigid telescoping member called the " flying boom," which is hydraulically actuated and has a small conical receptacle fitted at the trailing end. About four feet inboard of this receptacle are mounted the " ruddervators," two short aerofoil section vanes set symmetrically at an angle of about 130 degrees to each other, whose incidence can be varied by the boom operator. The boom is controlled by the boom operator who is situated in a transparent blister under the tail unit of the tanker at the root of the boom. It can be exended and rotated through a contact envelope similar in shape to that of the probe and drogue method.

The operating procedure is as follows. The boom operator talks, or signals, the receiver aircraft on to the extended boom until the boom end and contact point are only a few feet apart and then the two aircraft maintain the same relative position while the boom operator actually flies the boom into the coupling point, using the ruddervators and the telescoping action. The aircraft which use this system, *e.g.* the giant B-52 Strato fortresses, have their fuel coupling points well behind the cockpit, so that the pilot cannot actually see the contact being made. This is the fact that makes the boom operator's rôle necessary in order to make the final adjustments on the boom. The aircraft's coupling point is inside a hatch which opens to receive the boom end. The boom is located automatically, but the operator locks the coupling and starts the fuel flow. A series of switches mounted on a pedestal between the pilot and co-pilot of the tanker control the flow of the fuel from the wing storage tanks, through the main fuel tanks and thence to the boom. The flight engineer ensures that the remaining fuel is always correctly proportioned in the aircraft's various storage tanks.

Disconnection of the two aircraft may be initiated by the boom operator, who can unlock and retract the boom: by the pilot of the receiver, who can slow his aircraft down, causing the boom to unlock and the fuel flow to cease automatically; or by a fully automatic high fuel pressure disconnecting switch. In normal flight the boom fits flush under the tanker's tail unit causing only a minimum of drag.

The aircraft used as tankers for this system are the KC-97, a military version of the well-proven Stratocruiser, and the much newer KC-135, a military version of the Boeing 707, having a transferable fuel capacity of some 28,000 U.S. gallons.

14. The " Buddy Package "

Another method of establishing contact between the tanker and the receiver that can be applied to the latest types of combat aircraft is the " Buddy Package " developed by an American firm. Also designed to minimise air speed and altitude compatibility problems, this method utilises the probe and drogue principle, but has a retractable folding pipe instead of the conventional hose. The unit can easily be fitted to any combat aircraft capable of carrying a reasonable amount of fuel, being located either under the tail unit or in a streamlined fairing on the wing tip. The procedure is similar to that used in the probe and drogue method, the extending and retracting of the drogue being actuated by an operator in the tanker.

In all the methods described, complete radio silence can be maintained throughout the refuelling operation.

Conclusion

The first two methods described, Probe and Drogue, and Flying Boom are now in operational service. A small, compact hose reel and drogue unit can be fitted to any of the V-bombers and new marks of these aircraft are appearing with probes fitted. Canberras can be operated as a tanker/receiver team, and at Farnborough in 1958 a pair of Valiants gave a demonstration of probe and drogue by flying above the main runway actually transferring fuel from tanker to receiver.

On the other side of the Atlantic, Flying Boom has been developed on the usual grand American scale. A total of 888 KC-97s of various sub-types, were delivered to the U.S.A.F. and these are now being steadily replaced by a batch of 257 KC-135 Stratotankers, the first of which was placed in operational service with Strategic Air Command at Castle A.F.B., California, in January 1957. The U.S. Navy also use aerial refuelling techniques, having several North American AJ-2s converted to probe and drogue for refuelling carrier-borne Chance Vought F7U-3s. The compact A-16 hose reel and drogue unit, developed by Flight Refuelling Ltd., has been extensively adopted by the U.S.A.F. for its fighter aircraft, and 3-point tankers are standard equipment.

In spite of the great advances and improvements in the technique of flight refuelling, all of which have been tried and proved by military aircraft, the commercial aviation world has not adopted the idea at all. I think it is right to say that there are no regular airline services in the world using aerial refuelling. This is most surprising, since the early test on the North Atlantic and Bermuda runs proved conclusively the value of being able to refuel without landing. Some objection has been raised to the effect that passengers on the receiver aircraft would be able to see some of the operation taking place, this being an undesirable feature, but a technique could be developed whereby the receiver trailed the drogue and the tanker formated on it so that all activity would be behind the passenger.

Although it now seems unlikely that commercial aviation will adopt flight refuelling on scheduled services, this does not in any way reflect on the tremendous achievements made in developing the flight refuelling technique from a " hit and miss " air carnival stunt to an everyday routine operation.

TECHNICAL NOTES

The Three-Dimensional Turbulent Boundary Layer

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AN EXPERIMENTAL VERIFICATION of Coles' postulate of the Law of the Wake in three dimensions has been attempted. This hypothesis, which is strongly supported by experimental evidence in two dimensions, purports to represent the velocity vector in a yawed boundary layer by the sum of two vectors, whose magnitudes are found from the universal law of the wall, and a second universal function, the law of the wake. It has been found that the velocity vector can be represented in this way and the wake function required for this is very similar to Coles' form. Limitations of the experimental arrangement however prohibited the comparison of vector directions with that proposed by Coles.

NOTATION

C	undetermined coefficient
K	surface roughness (non-dimensional)
$\mathbf{q}$	vector quantity$=\mathbf{u}/u^*$
$\mathbf{q}^*$	unit vector in direction of wall shear stress
U	free stream velocity
u	velocity
u^*	friction velocity$=\sqrt{(\tau_0/\rho)}$
$\mathbf{u}$	velocity vector
w	universal function; the Coles wake function
w_h	unnormalised component AC
w_j	unnormalised component CB
w_H	curve derived from AC component, *see* Fig. 2
w_J	curve derived from CB component, *see* Fig. 3
y	distance from surface
θ	angle such that $\tan\theta=w_j/w_h$
κ	undetermined coefficient
μ	fluid viscosity
ν	kinematic viscosity
ρ	density
ϕ	yaw angle

INTRODUCTION

Since the full equations of motion for turbulent flow, involving the Reynold's stress terms, are at the moment insoluble, other less rigorous methods of solution must be found if prediction of boundary layer behaviour is to be made. One such approach is the use of dimensional analysis, a practice well established in aerodynamics for evaluating problems which defy exact solution. For this purpose, it is usual to consider the boundary layer in two broad parts:—

(*i*) a region near the surface where the effects of viscosity are still evident

(*ii*) a region well away from the surface, where the flow properties are determined by the turbulent movement.

Near the surface, the relevant variables affecting the velocity, u, are the distance from the surface, y, the surface shear stress τ_0, the surface roughness K and the fluid properties viscosity μ and density ρ. Well away from the surface, the effect of viscosity and roughness may be neglected, but the free stream velocity U at the edge of the boundary layer, distant δ from the surface, will be applicable. By the conventional methods these two sets of variables lead to

$$u/u^*=f(yu^*/\nu,\ K) \quad . \quad . \quad . \quad (1)$$

$$(U-u)/u^*=g(y/\delta) \quad . \quad . \quad . \quad (2)$$

where

$u^*=\sqrt{\left(\frac{\tau_0}{\rho}\right)}$, the friction velocity, and ν is the kinematic viscosity.

The first equation is known as the law of the wall, the second as the velocity defect law.

A rather elegant analysis by Millikan for aerodynamically smooth walls[1] considering some region of the boundary layer where both these laws are true, leads to another equation

$$u/u^*=(1/\kappa)\log_e yu^*/\nu+C+h(y/\delta) \quad . \quad (3)$$

wherein the functional relationship of the law of the wall has been replaced by an explicit equation, with an added function, which is a function of y only, and which may be regarded as "correcting" the logarithmic form for the turbulent effects. κ and C are undetermined coefficients.

The function $h(y/\delta)$ is obviously related to the velocity defect law, and Coles has shown[2] that this may be written in the form

$$h(y/\delta)=\Pi(x)\,w(y/\delta)$$

where $\Pi(x)$ is some undetermined function of x the distance along the surface, and w is a universal function (*i.e.* applicable to all turbulent flows) whose shape must be determined experimentally. This function Coles calls the wake function.

There is ample evidence of the truth of the logarithmic form in turbulent boundary layer profiles. From plots of well attested experiments, by subtracting the logarithmic part from the measured profile, Coles derives a form for the wake function which is roughly sinusoidal in shape. This standard form is altered by the scale factor Π to bring the ordinates up to that required to fit any particular case.

WAKE FUNCTION IN THREE DIMENSIONS

The apparent success of the wake concept in two dimensions leads Coles to propose a model for three-dimensional flows, of the form

$$\mathbf{q}=\mathbf{q}^*f(yu^*/\nu)+\mathbf{q}^*\Pi w(y/\delta) \quad . \quad . \quad (4)$$

where $\mathbf{q}$ is a vector quantity$=\mathbf{u}/u^*$ and $\mathbf{q}^*$ the unit vector in the direction of the wall shear stress. The law of the wall contribution is the logarithmic form described above with direction the same as the wall shear stress, and w having magnitude defined by the wake function and direction such as to close the vector triangle. The combination $\mathbf{q}^*\Pi$ then has a different direction to that of $\mathbf{q}^*$, *i.e.* Π has tensorial properties. (*See* Fig. 1.) Coles has suggested this direction might be that of the pressure gradient at the point.

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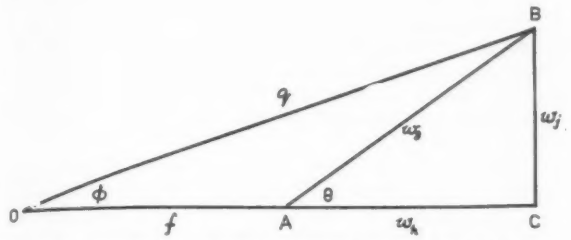

FIGURE 1. Resolution of velocity vector into wall (f) and wake (w_g) components.

PRESENT EXPERIMENTS

The present experiments were therefore designed to test this hypothesis. Measurements of the profile of velocity and yaw were made near the trailing edge of a delta wing model, using a hot wire anemometer for the velocity and a small cobra probe for the yaw traverses. Runs were made at two stations, using two free stream velocities and three angles of incidence of the wing, giving twelve sets of readings in all. The resulting profiles were analysed by resolving the velocity measured into two components, parallel and normal to the flow direction at the surface namely *OC*, *CB* respectively. Using the component *OC*, a fit was made to the logarithmic law of the wall, and the resulting profile subtracted from the measured profile to give the wake contribution *AC*. The normal component *CB* was entirely of wake form. The ordinates of these wake components *CB*, *AC* were then multiplied by an appropriate factor to bring the maximum value to 2. The results are plotted in Figs. 2 and 3, w_H being the curve derived from the *AC* component, and w_J from the normal *CB* component.

DISCUSSION

It is observed that the correlation for the w_j component is much higher than that of the w_h component. Using a least squares method, the curve shown was fitted to these

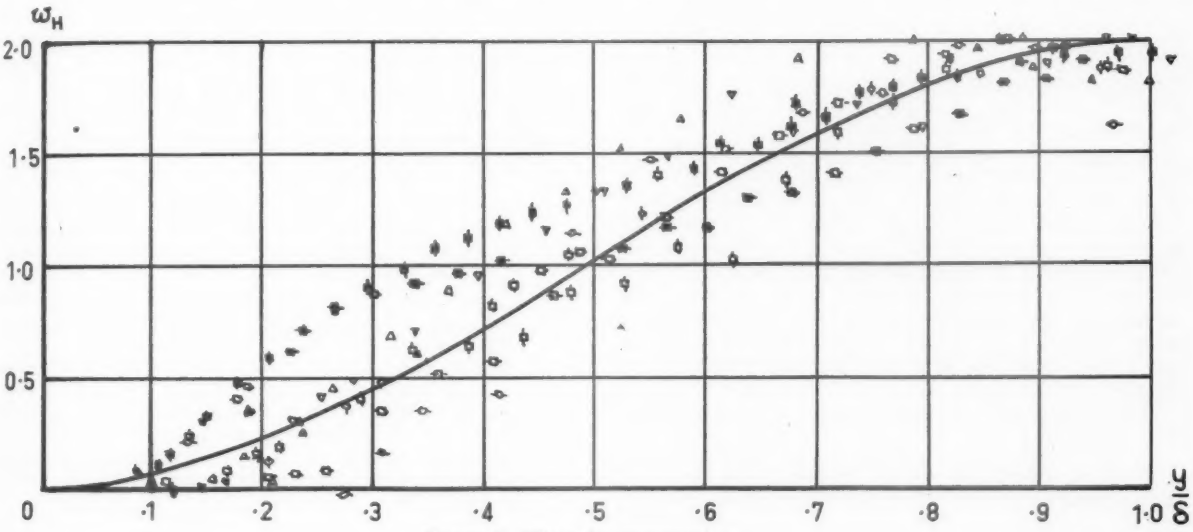

FIGURE 2. The wake component w_H.

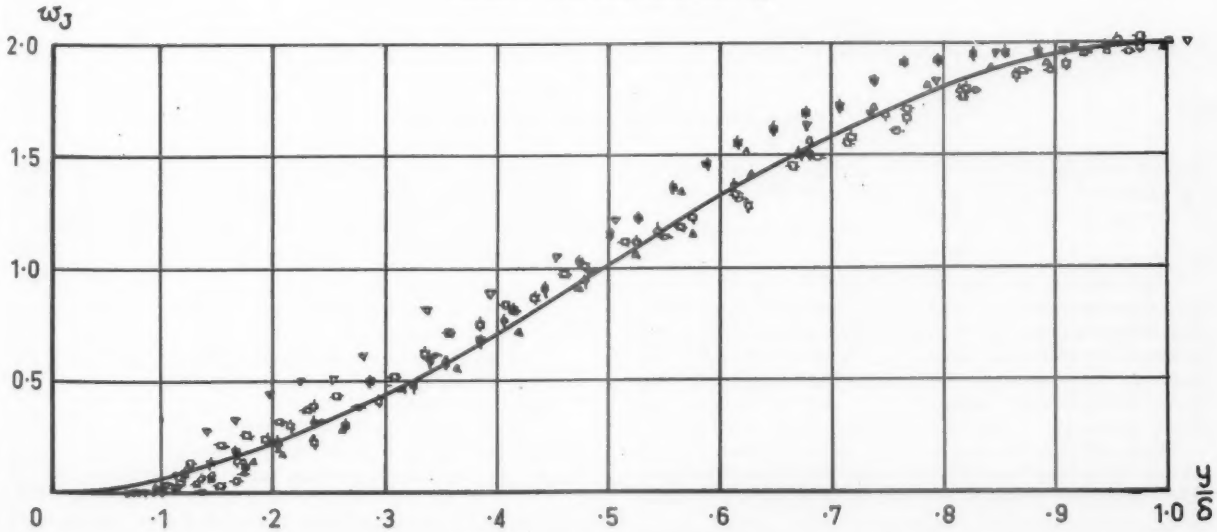

FIGURE 3. The wake component w_J.

TABLE I

y/δ	*Coles' values of w*	*Present Experiment values of w*
0	0	0
0·05	0·004	0·014
0·10	0·029	0·053
0·15	0·084	0·120
0·20	0·168	0·206
0·25	0·272	0·316
0·30	0·396	0·439
0·35	0·535	0·579
0·40	0·685	0·723
0·45	0·838	0·876
0·50	0·994	1·028
0·55	1·152	1·189
0·60	1·307	1·341
0·65	1·458	1·486
0·70	1·600	1·615
0·75	1·729	1·729
0·80	1·840	1·824
0·85	1·926	1·900
0·90	1·980	1·955
0·95	1·992	1·988
1·00	2·000	2·000

points. A comparison of the ordinates of this curve and Coles proposed form is given in Table I, and it is seen that the agreement is quite good.

It was felt that although the scatter for the component w_h was much higher, these points also supported the notion of the wake function. For the form of the curve derived depends on the accuracy of the evaluation of u^* in fitting to the logarithmic law† whereas only the magnitude but not the shape of the w_j component is affected in this way. Moreover, if Coles theory is correct w_j and w_h are merely components of some other vector w_0 so that both will have the same form being related to this vector. Hence, if w_h and w_j are components of w_0 all three must have the same shape.

The questionable value of u^*, and the small angles of yaw experienced in the experiment ($\phi < 5°$) makes an accurate determination of the angle θ difficult. Even if a better value could have been found, comparison with the pressure gradient direction, as suggested by Coles would still be difficult due to the very small pressures and hence indeterminate pressure gradient on this part of the wing (*see* Ref. 3).

An attempt was made to fit the data to the flow model proposed by Johnstone[4], which is simpler than Coles model. In this, the tip of the velocity vector **u** lies on a straight line making some direction γ with the direction of the surface shear stress. However, the attempt was not successful.

† or, *e.g.* if the edge of the sub-layer is taken at $\frac{yu^*}{\nu} = 30$ instead of 50.

CONCLUSION

For the particular experimental set-up concerned, on which there existed a yawed turbulent boundary layer, it was found that the velocity profile could be represented by two vectors, one with direction of the surface shearing stress and of magnitude given by the logarithmic law of the wall, and another with magnitude given by a function very close to Coles wake function in two dimensions, and direction such that the vector triangle between the velocity and law of the wall closed. Although it was not possible to relate the direction of the wake function to the flow properties, it was felt that the shape of the wake vector affords support for Coles hypothesis of the law of the wake in three dimensions.

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Shear Flow Past a Wing of Circular Plan Form

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THE PROBLEM of flow past a thin wing of circular plan form when the undisturbed flow is along the x-axis, $U(z) = Az + V$ has been solved.

1. INTRODUCTION

The solutions for potential flow past a wing of circular plan form have been given in terms of spheroidal co-ordinates using the concept of acceleration potential[1,2]. The solutions can be easily adapted to the case when the undisturbed flow is weakly sheared. In the particular case when the undisturbed velocity $U(z) = Az + V$, where A and V are constants, the pressure equation can be simplified by a simple substitution and the solution for a thin wing of circular plan form can be easily constructed from the solution for potential flow[2]. If the wings in the two cases develop the same lift, it is found that the wing in shear flow should be slightly cambered (potential wing is uncambered). The results are shown graphically in Fig. 1.

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NOTATION

A	constant vorticity
c	a small constant
$f(\tau)$	function of τ
$F(S)$	real valued continuous function of S
$g(Y)$	function of Y defined in (31)
$h(x, y)$	function of x, y
$l(y)$	spanwise lift distribution
$N = ARC/V$,	a non-dimensional parameter
p, p_{+0}, p_{-0}	pressure, pressure for $z = +0$, pressure for $z = -0$
P_n, Q_n	Legendre's functions of the first and second kind

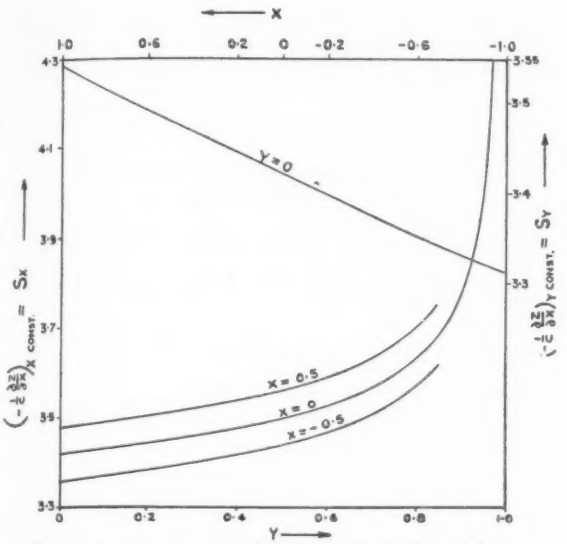

FIGURE 1. Spanwise and chordwise distribution of slope.

R radius of the wing
Re real part
S_z, S_y spanwise, chordwise distributions of local slope
(r, θ, z) cylindrical co-ordinates
(u_1, v_1, w_1) Cartesian induced velocity components
$U(z)$ undisturbed shear flow along x-axis
V constant velocity
x, y, z Cartesian co-ordinates
$X, Y, Z = x/R,\ y/R,\ z/R$
μ, ν, ζ spheroidal co-ordinates
λ, k integers (0, 1, 2, ...)
$\phi(x, y)$ function of x, y
ρ density

$$\tau = \sqrt{\left[\frac{1-\mu^2}{1+\nu^2}\right]}\, e^{\zeta}$$

$$\xi = \cos^{-1}\frac{R}{r}$$

Ω, Ω' functions of x, y, z
$\Omega'_{+0}, \Omega'_{-0}$ values of Ω' for $z = +0$ and for $z = -0$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

Γ gamma function

2. EQUATIONS OF MOTION AND SOLUTION

The linearised equations for the steady inviscid incompressible motion, when the undisturbed velocity $U(z)$ along the axis of x is a function of z and disturbances are small, are

$$U\frac{\partial u_1}{\partial x} + w_1\frac{dU}{dz} = -\frac{1}{\rho}\frac{\partial p}{\partial x}, \quad . \quad . \quad (1)$$

$$U\frac{\partial v_1}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial y}, \quad . \quad . \quad . \quad (2)$$

$$U\frac{\partial w_1}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial z}, \quad . \quad . \quad . \quad (3)$$

The equation of continuity is

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0. \quad . \quad . \quad (4)$$

From (1)—(4) the equation for pressure is

$$\nabla^2 p - \frac{2}{U}\frac{dU}{dz}\frac{\partial p}{\partial z} = 0. \quad . \quad . \quad (5)$$

The components of the induced velocity are

$$u_1 = -\frac{1}{U}\frac{dU}{dz}\int_{-\infty}^{x} w_1 dx - \frac{p}{\rho U}, \quad . \quad (6)$$

$$v_1 = -\frac{1}{\rho U}\int_{-\infty}^{x}\frac{\partial p}{\partial y}dx, \quad . \quad . \quad . \quad . \quad (7)$$

$$w_1 = -\frac{1}{\rho U}\int_{-\infty}^{x}\frac{\partial p}{\partial z}dx, \quad . \quad . \quad . \quad . \quad (8)$$

provided that the pressure is zero when $x \to -\infty$.

Consider a thin and slightly cambered wing of circular plan form ($z = 0$, $r \leqslant R$) placed in a weakly sheared undisturbed flow $U(z)$ along the x-axis. The spanwise lift distribution is

$$l(y) = -2\int_{-\sqrt{(R^2-y^2)}}^{\sqrt{(R^2-y^2)}} p_{z=0}\, dx \quad . \quad . \quad (9)$$

The pressure p has to satisfy the boundary conditions:

(*i*) The disturbance vanishes at infinities:
$p = 0$, for $|x|, |y|, |z| \to \infty$.

(*ii*) $p_{+0} = p_{-0}$ on $z = 0$, $R < r$: continuity of pressure outside the wing surface on the plane $z = 0$.

(*iii*) $p = -p(x, y)$ on $z = +0$, $r < R$,
$p = +p(x, y)$ on $z = -0$, $r < R$,
p is negative on the upper surface of the wing and it is positive on its lower surface.

(*iv*) $\dfrac{\partial p}{\partial z}$ should be continuous on the plane $z = 0$:
continuity of the induced downwash velocity, w_1, on the plane $z = 0$.

(*v*) p is infinite at the leading edge (as in potential flow) and it is zero at the trailing edge in accordance with the Joukowski condition.

(*vi*) The component of the velocity normal to the wing surface is zero:
$[w_1]_{z=0} = U(z)\dfrac{\partial z}{\partial x}$, where $z = h(x, y)$ is the equation of the wing surface.

For a particular type of shear flow $U(z) = Az + V$, equation (5) becomes

$$\nabla^2 p - \frac{2A}{Az+V}\frac{\partial p}{\partial z} = 0. \quad . \quad . \quad (10)$$

Putting $$\frac{p}{\rho} = \left(1 + \frac{Az}{V}\right)\Omega - \frac{A}{V}\int_0^z \Omega dz, \quad . \quad (11)$$

equation (10) simplifies to

$$\nabla^2\Omega = \frac{A}{V}\frac{\partial\Omega}{\partial z}\bigg|_{z=0}. \quad . \quad . \quad (12)$$

The case $A = 0$, corresponds to potential flow past the wing[2] and Ω then represents the "acceleration potential", and (12) reduces to Laplace's equation. If $\Omega = \Omega' + \phi(x, y)$, (12) becomes $$\nabla^2\Omega' = 0 \quad . \quad . \quad . \quad . \quad (13)$$

$$\nabla_1^2\phi = \frac{A}{V}\frac{\partial\Omega'}{\partial z}\bigg|_{z=0}. \quad . \quad . \quad (14)$$

In virtue of the condition (*iii*) $\phi(x, y)$ must be zero for $r \leqslant R$ and $\Omega'_{+0} = -\Omega'_{-0}$ on $z = 0$, $r < R$. It follows from (14) that $\dfrac{\partial\Omega'}{\partial z}\bigg|_{z=0} = 0$ when $r < R$. Then (*ii*) can be

satisfied only when $\Omega'_{+0}=\Omega'_{-0}=0$ on $z=0$, $r>R$. The conditions (*ii*), (*iii*), (*iv*), (*v*) are satisfied if Ω' is taken as Kinner's solution[2] for the potential flow.

Thus the solution of equation (13) may be taken as

$$\Omega'=\frac{\mu}{\mu^2+\nu^2}\,Re\,[f(\tau)], \quad . \quad . \quad (15)$$

where, $x=R\sqrt{(1-\mu^2)}\sqrt{(1+\nu^2)}\cos\zeta$,
$y=R\sqrt{(1-\mu^2)}\sqrt{(1+\nu^2)}\sin\zeta$,
$z=R\mu\nu$.

Since $\phi(x, y)=0$ on $r=R$, the conditions at the leading and trailing edges are satisfied if, for $Re(\tau)>0$,

$$f(\tau)=\frac{1}{\pi}\int_{-1}^{1}F(S)\left[\frac{\tau^{-1}}{1+\tau^{-2}S^2}-\frac{\tau}{1+\tau^2S^2}\right]dS, \quad (16)$$

where $F(S)$ is any continuous real valued function of S which is defined in the closed interval $<-1, 1>$.

For $F(S)=(-S^2)^k$, $k=0, 1, 2, \ldots$, (16) becomes

$$f(\tau)=\tau^{2k}-\frac{2}{\pi}(-1)^k\sum_{\lambda=0}^{\infty}(-1)^\lambda\left(\frac{1}{2\lambda+2k+1}+\frac{1}{2\lambda-2k+1}\right)\tau^{2\lambda+1} \quad . \quad . \quad (17)$$

which with (15) gives

$$\Omega'=\frac{\mu}{\mu^2+\nu^2}\,Re\left[\tau^{2k}-\frac{2}{\pi}(-1)^k\sum_{\lambda=0}^{\infty}(-1)^\lambda\left(\frac{1}{2\lambda+2k+1}+\frac{1}{2\lambda-2k+1}\right)\tau^{2\lambda+1}\right] \quad . \quad (18)$$

From (18)

$$\left.\frac{\partial\Omega'}{\partial z}\right|_{z=0}=0, \quad r\leqslant R,$$

$$=\frac{R^2}{(r^2-R^2)^{3/2}}\,Re\left[f\left(\frac{R}{r}\,e^{i\theta}\right)\right], \quad r\geqslant R, \quad . \quad (19)$$

where r, θ are the plane polar co-ordinates. The solution of (14) gives

$$\phi(x, y)=0, \quad r\leqslant R$$

$$=-\frac{AR}{V}\cos 2k\theta\left[\left(\frac{r}{R}\right)^{2k}\int_{\pi/2}^{\xi}\cos^{4k}\xi\,d\xi+\frac{\Gamma[(4k+1)/2]\,\Gamma\frac{1}{2}}{2\Gamma(2k+1)}\left(\frac{R}{r}\right)^{2k}\right]$$

$$+\frac{2AR}{\pi V}(-1)^k\left[\sum_{\lambda=0}^{\infty}(-1)^\lambda\left(\frac{1}{2\lambda+2k+1}+\frac{1}{2\lambda-2k+1}\right)\times\right.$$

$$\left.\times\cos(2\lambda+1)\theta\left\{\left(\frac{r}{R}\right)^{2\lambda+1}\int_{\pi/2}^{\xi}\cos^{4\lambda+2}\xi\,d\xi+\right.\right.$$

$$\left.\left.+\frac{\Gamma(2\lambda+3/2)\,\Gamma\frac{1}{2}}{2\Gamma(2\lambda+2)}\left(\frac{R}{r}\right)^{2\lambda+1}\right\}\right], \quad r\geqslant R. \quad . \quad (20)$$

Thus the pressure is given by

$$\frac{p}{\rho}=\left(1+\frac{Az}{V}\right)\Omega'-\frac{A}{V}\int_0^z\Omega'dz, \text{ for } r\leqslant R. \quad . \quad . \quad (21)$$

$$\frac{p}{\rho}=\left(1+\frac{Az}{V}\right)\Omega'-\frac{A}{V}\int_0^z\Omega'dz+\phi(x, y), \text{ for } r\geqslant R. \quad (22)$$

It can be seen from (22) that

$$p\to 0 \quad \text{as} \quad |x|, \ |y|\to\infty,$$

but $\frac{p}{\rho}\to\frac{A}{V}\left.\frac{\partial\Omega'}{\partial z}\right|_{z=0}+\phi(x, y)$=a function of (x, y), as $|z|\to\infty$. The condition (*i*) is, therefore, not satisfied completely. This discrepancy in pressure at $|z|=\infty$ is due to the particular choice of the undisturbed velocity.

From (8) the downwash velocity on the plane $z=0$ is

$$w_1|_{z=0}=\frac{(-1)^k}{V}\frac{d}{dY}\left[Q_{2k}(Y)+\sum_{\lambda=0}^{\infty}\left(\frac{1}{2\lambda+2k+1}+\frac{1}{2\lambda-2k+1}\right)P_{2\lambda+1}(Y)\right]. \quad (23)$$

The spanwise lift distribution from (9) is

$$l(y)=2\pi\rho R(-1)^{k+1}P_{2k}(Y) \quad . \quad . \quad (24)$$

The local slope of the wing is given by (*vi*) to be

$$\frac{\partial z}{\partial x}=\frac{(-1)^k}{V(V+Az)}\frac{d}{dY}\left[Q_{2k}(Y)+\sum_{\lambda=0}^{\infty}\left(\frac{1}{2\lambda+2k+1}+\frac{1}{2\lambda-2k+1}\right)P_{2\lambda+1}(Y)\right]. \quad (25)$$

In terms of the lift distribution $l(y)$[1],

$$\frac{\partial z}{\partial x}=\frac{1}{V(V+Az)}\frac{d}{dY}\left[\int_{-1}^{1}F(S)\,H(S, Y)\,dS-\right.$$

$$\left.-\frac{1}{2\pi\rho R}\int_{-1}^{1}\frac{l(RY')}{Y-Y'}\,dY'\right] \quad . \quad (26)$$

where $F(S)=-\frac{1}{2\pi\rho R}\int_{-1}^{1}l(RY)\frac{(1-S^2)\,dY}{2(1-2YS+S^2)^{3/2}}$, (27)

and $H(S, Y)=\begin{cases}\dfrac{S^{-1}+1}{2\sqrt{(1-2SY+S^2)}}, & S<0\\[2ex] \dfrac{S^{-1}-1}{2\sqrt{(1-2SY+S^2)}}, & S>0\end{cases}$ . . . (28)

Thus if $l(y)$ is given, the shape of the wing can be found from (26).

3. EXAMPLE

Let the spanwise lift distribution $l(y)$ be given by

$$l(y)=l(RY)=2\pi\rho RV^2c\sqrt{(1-Y^2)}. \quad . \quad (29)$$

From (27)

$$F(S)=-V^2c\left(\frac{1-S^2}{S}\right)(K(S)-E(S)), \quad . \quad . \quad (30)$$

where, $K(S)=\int_0^1\frac{dY}{\sqrt{[(1-Y^2)(1-S^2Y^2)]}}$

$$E(S)=\int_0^1\left[\frac{1-S^2Y^2}{1-Y^2}\right]^{\frac{1}{2}}dY,$$

Substituting from (29) and (30) in (26), one gets

$$\frac{\partial z}{\partial x}=-\frac{cV}{V+Az}\left\{\int_0^1\frac{(1-S)^2(1+S)}{2S^2}(K(S)-E(S))\times\right.$$

$$\left.\times\left(\frac{1}{(1-2SY+S^2)^{3/2}}+\frac{1}{(1+2SY+S^2)^{3/2}}\right)dS+\pi\right\}$$

$$=-\frac{cV}{V+Az}\,g(Y). \quad . \quad . \quad . \quad . \quad (31)$$

Integrating (31) and applying the boundary condition, $x=0$ at $z=0$, one finds,

$$z=-\frac{V}{A}\pm\frac{V}{A}\left(1-2\frac{Acx}{V}g(Y)\right)^{1/2}$$

$$=-\frac{V}{A}+\frac{V}{A}\left(1-2N\frac{x}{R}g(Y)\right)^{1/2} \quad . \quad . \quad (32)$$

The minus sign has been rejected as it does not give the potential case when $A\to 0$.

TABLE I

SPANWISE AND CHORDWISE SLOPES FOR $N=0{\cdot}01$

Y	$-\frac{1}{c}\frac{\partial Z}{\partial X}$			X	$-\frac{1}{c}\frac{\partial Z}{\partial X}$
	$X=0$	$X=-0{\cdot}5$	$X=0{\cdot}5$		$Y=0$
0	3·42	3·36	3·48	−1·0	3·309
0·3	3·46	3·40	3·52	−0·8	3·330
0·6	3·53	3·47	3·59	−0·6	3·352
0·8	3·63	3·57	3·70	−0·4	3·374
0·825	3·65	3·59	3·72	−0·2	3·397
0·850	3·68	3·62	3·75	0·0	3·420
0·875	3·72			0·2	3·444
0·9	3·77			0·4	3·468
0·925	3·85			0·6	3·492
0·950	4·00			0·8	3·518
0·975	4·40			1·0	3·543
1·0	∞				

From (32)

$$-\frac{1}{c}\frac{\partial Z}{\partial X}=g(Y)\left[1+NXg(Y)+\frac{3}{2}N^2X^2g^2(Y)+\right.$$
$$\left.+\frac{5}{2}N^3X^3g^3(Y)+\ldots\right].$$

The numerical values of $-\frac{1}{c}\frac{\partial Z}{\partial X}$ at different sections for $N=0{\cdot}01$, are given in Table I. The spanwise and chordwise distributions of the local slope are presented graphically (Fig. 1). The local slope in the spanwise direction varies very slowly with Y except near the tips where it becomes finite; and the chordwise slope varies with X while it is independent of X in potential flow. This shows that the wing in shear flow is slightly cambered.

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The Natural Frequencies of Free and Constrained Non-Uniform Beams

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1. INTRODUCTION

The Rayleigh-Ritz method is well known as an approximate method of determining the natural frequencies of a conservative system, using a constrained deflection form. On the other hand, if a general deflection form (*i.e.* an unconstrained form) is used, the method provides a theoretically exact solution. An unconstrained form may be obtained by expressing the deflection as an expansion in terms of a suitable set of orthogonal functions, and in selecting such a set, it is convenient to use the known normal modes of a suitably chosen " basic system." The given system, whose vibration properties are to be determined, can then be regarded as a " modified system," which is derived from the basic system by a variation of mass and elasticity. A similar procedure has been applied to systems with a finite number of degrees of freedom[1]. In the present note the method is applied to simple non-uniform beams, and to beams with added masses and constraints. A concise general solution is obtained, and an iteration process of obtaining a numerical solution is described.

2. VIBRATION OF A NON-UNIFORM BEAM

In determining the natural frequencies and modes of a non-uniform beam, a basic system may be chosen consisting of a uniform beam having the same length and end conditions.

Provided these conditions are simply supported, free, or clamped, then the normal modes of the uniform beam will satisfy the end conditions of the non-uniform beam. Using co-ordinates x and y to denote distance along the beam and transverse deflection respectively, any deflection of the non-uniform beam (*i.e.* the modified system) may be expressed in the form

$$y(x)=\sum_{r=1}^{\infty}\phi_r y_r(x) \quad . \quad . \quad . \quad (1)$$

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where the functions $y_r(x)$ are the normal modes of the basic system, and the quantities ϕ_r are the normal co-ordinates of y referred to the basic system.

If the modified system is executing free vibration of circular frequency ω, then, by equating the maximum kinetic and potential energies, we have

$$\omega^2\int_0^l my^2dx-\int_0^l (y'')^2B\,dx=0. \quad . \quad . \quad (2)$$

In equation (2) l, $m(x)$, and $B(x)$, are the length, mass per unit length, and flexural rigidity of the non-uniform beam, and primes denote differentiation with respect to x.

By Rayleigh's principle, $\partial\omega/\partial\phi_s=0$, $s=1, 2, \ldots$ and by differentiating (2) with respect to ϕ_s, we obtain

$$\omega^2\int_0^l myy_s\,dx-\int_0^l By''y_s''dx=0 \quad . \quad . \quad (3)$$

and by substituting from (1)

$$\omega^2\int_0^l\{my_s\sum_r\phi_r y_r\}\,dx-\int_0^l\{By_s''\sum_r\phi_r y_r''\}\,dx=0\,. \quad (4)$$

Equations (4) yield an infinite set of linear equations which may be written in the form

$$\omega^2\begin{bmatrix}a_{11} & a_{12} & \cdot\cdot\\ a_{21} & a_{22} & \cdot\\ \cdot\cdot & \cdot\cdot & \cdot\cdot\end{bmatrix}\begin{bmatrix}\phi_1\\ \phi_2\\ \cdot\cdot\end{bmatrix}-\begin{bmatrix}b_{11} & b_{12} & \cdot\cdot\\ b_{21} & b_{22} & \cdot\cdot\\ \cdot\cdot & \cdot\cdot & \cdot\cdot\end{bmatrix}\begin{bmatrix}\phi_1\\ \phi_2\\ \cdot\cdot\end{bmatrix}=0 \quad . \quad (5)$$

where $a_{rs}=\int_0^l my_ry_s\,dx$, $b_{rs}=\int_0^l By_r''y_s''dx$.

In practice, it will be necessary to terminate the series (1) after a finite number of terms. Equations (5) are then analogous to those governing the free vibration of a system

with finite freedom, having an inertia matrix $[a_{rs}]$, and a stiffness matrix $[b_{rs}]$. While the equations may be solved by the usual methods applicable to such systems, in many cases a numerical solution by successive approximation may be used, which will be described below.

It is well known that the functions y_r satisfy the orthogonal properties

$$\int_0^l m_0 y_r y_s dx=0, \quad \int_0^l B_0 y_r'' y_s'' dx=0 \qquad \text{if } r\neq s \quad (6)$$

where m_0 and B_0 are the mass per unit length and flexural rigidity of the basic system. If the mass and stiffness distributions of the modified system differ only slightly from those of the basic system, then the matrix elements a_{rs}, b_{rs} are small if $r\neq s$. Also, the functions y_r may be regarded as first approximations to the modes of the modified system. To determine the r^{th} mode and frequency, therefore, we take $\phi_r=1$ and treat ϕ_s $(s\neq r)$ as a small quantity. Putting $\phi_r=1$, the r^{th} and s^{th} equations of (5) may be written

$$\omega^2 a_{rr}-b_{rr}+\sum_{i\neq r}\phi_i(\omega^2 a_{ri}-b_{ri})=0. \quad . \quad . \quad (7)$$

$$\phi_s(\omega^2 a_{ss}-b_{ss})+(\omega^2 a_{sr}-b_{sr})+\sum_{\substack{i\neq r\\ i\neq s}}\phi_i(\omega^2 a_{si}-b_{si})=0. \quad (8)$$

In the first approximation we neglect the products of small quantities. The terms under the summation signs may then be ignored, and the equations give directly the first approximations for ω and ϕ_s. Substituting these values for ϕ_s in the previously neglected terms in (7) and (8), second approximations may be obtained, and the process may be repeated.

This iteration method, though based on the assumption of small differences between the modified and basic systems, may often be used successfully in cases where the mass and stiffness distributions of the two systems differ considerably. The method has the advantage that any of the higher frequencies and modes may be calculated directly, without first calculating the lower modes. In applying the method, it is usually sufficient to consider only a small number of terms of the series (1) during the early stages, additional terms being taken into account as the approximation proceeds.

As a numerical example, consider a simply-supported beam of uniform thickness, but tapered in plan, as shown in Fig. 1. The mass and flexural rigidity of the beam are given by

$$m=m_0\left(1+\frac{2x}{l}\right), \quad x<l/2$$

$$B=B_0\left(1+\frac{2x}{l}\right), \quad x<l/2$$

where $m_0=\rho bh$, $B_0=Ebh^3/12$, ρ being the density and E Young's modulus. The appropriate basic system is a simply-supported uniform beam, whose normal modes are

$$y_r=\sin\frac{r\pi x}{l}.$$

Taking initially $\phi_1=1$, and writing $\lambda=\omega^2 m_0 l^4/B_0\pi^4$, the following successive approximations are obtained for the fundamental frequency and mode

	ϕ_1	ϕ_3	ϕ_5	ϕ_7	λ
(1)	1	—	—	—	1
(2)	1	0·01331	—	—	0·9873
(3)	1	0·01330	$0{\cdot}7\times10^{-4}$	—	0·9875
(4)	1	0·01338	$1{\cdot}49\times10^{-4}$	$2{\cdot}96\times10^{-4}$	0·9872

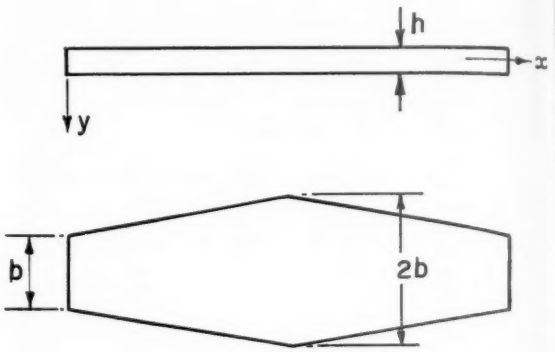

FIGURE 1.

For the next symmetrical mode and frequency we obtain

	ϕ_1	ϕ_3	ϕ_5	ϕ_7	λ
(1)	—	1	—	—	81
(2)	0·1071	1	—	—	82·04
(3)	0·1069	1	0·0355	—	81·31
(4)	0·1069	1	0·0361	0·0015	81·30

In this numerical example the iteration process shows good convergence.

3. BEAMS WITH CONSTRAINTS AND ADDED MASSES

Consider a beam with a transverse elastic constraint of stiffness k at the point $x=x_1$, and an added mass M at $x=x_2$, as shown in Fig. 2(*a*). The energy equation (2) now becomes

$$\omega^2\int_0^l my^2dx+M\omega^2[y(x_2)]^2-\int_0^l (y'')^2Bdx-k[y(x_1)]^2=0. \quad . \quad (9)$$

Again assuming for y the expansion (1), and differentiating (9) with respect to ϕ_s, we obtain

$$\omega^2\int_0^l myy_s dx+M\omega^2 y(x_2)\,y_s(x_2)-\int_0^l By''y_s''dx-ky(x_1)\,y_s(x_1)=0 \quad (10)$$

The solution may again be written in the form (5), with the following values for the matrix elements

$$\left.\begin{aligned} a_{rs}&=\int_0^l my_r y_s dx+My_r(x_2)\,y_s(x_2)\\ b_{rs}&=\int_0^l By_r''y_s''dx+ky_r(x_1)\,y_s(x_1)\end{aligned}\right\} \quad . \quad . \quad (11)$$

The method can obviously be extended to beams with several added masses and constraints.

If the beam has a spring-supported mass at the point $x=x_3$, as shown in Fig. 2(*b*), then it is necessary to introduce an additional co-ordinate for the deflection of the mass. The basic system in this problem can be taken as a uniform beam, together with an isolated mass (*i.e.* the case $k=0$). The deflection of the beam is given by equation (1), and the deflection of the mass will be denoted by ϕ_0. The energy equation becomes

$$\omega^2\int_0^l my^2dx-\int_0^l (y'')^2Bdx+M\omega^2\phi_0^2-k[\phi_0-y(x_3)]^2=0. \quad . \quad (12)$$

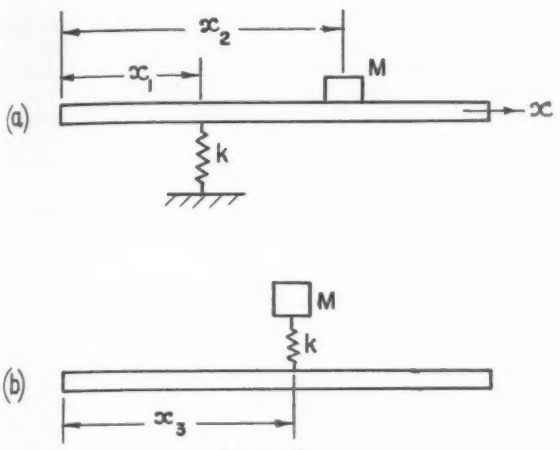

FIGURE 2.

Differentiating with respect to ϕ_0 we have

$$M\omega^2\phi_0 - k\,[\phi_0 - y\,(x_3)] = 0. \quad . \quad . \quad (13)$$

Differentiating with respect to ϕ_s, $s > 0$,

$$\omega^2 \int_0^l myy_s dx - \int_0^l By''y_s'' dx + k\,[\phi_0 - y\,(x_3)]\,y_s\,(x_3) = 0. \quad (14)$$

Equations (13) and (14) may be put in the form

$$\omega^2 \begin{bmatrix} a_{00} & a_{01} & \cdots \\ a_{10} & a_{11} & \cdots \\ \cdots & \cdots & \end{bmatrix} \begin{bmatrix} \phi_0 \\ \phi_1 \\ \cdots \end{bmatrix} - \begin{bmatrix} b_{00} & b_{01} & \cdots \\ b_{01} & b_{11} & \cdots \\ \cdots & \cdots & \end{bmatrix} \begin{bmatrix} \phi_0 \\ \phi_1 \\ \cdots \end{bmatrix} = 0 \quad . \quad . \quad (15)$$

where

$$a_{00} = M,\ b_{00} = k,\ a_{0r} = a_{r0} = 0.$$

$$b_{0r} = b_{r0} = -ky_r\,(x_3).$$

$$a_{rs} = \int_0^l my_r y_s dx. \quad . \quad . \quad . \quad (16)$$

$$b_{rs} = \int_0^l By_r''y_s'' dx + ky_r\,(x_3)\,y_s\,(x_3),\ (r > 0,\ s > 0).$$

As a numerical example, consider a uniform simply-supported beam, with a mass and spring attached at the mid-point. Also, let $M = ml$, the total mass of the beam, and let $k = 48B/l^3$, the transverse stiffness of the beam at the mid-point. This problem has been considered by Lee and Saibel[2].

To obtain an initial approximation, we neglect all the co-ordinates except ϕ_0 and ϕ_1. Equations (15) then give the two solutions

$$\phi_0 = 1, \qquad \phi_1 = 0{\cdot}5577, \qquad \lambda = 0{\cdot}2180$$

and

$$\phi_0 = -0{\cdot}2788, \quad \phi_1 = 1, \qquad \lambda = 2{\cdot}260$$

where $\lambda = m\omega^2 l^4/B\pi^4$. These solutions provide initial approximations for the first two symmetrical modes and frequencies. For the first mode, the iteration process gives the following results

	ϕ_0	ϕ_1	ϕ_3	ϕ_5	λ
(1)	1	0·5577	—	—	0·2180
(2)	1	0·5546	$-5{\cdot}37\times10^{-3}$	—	0·2168
(3)	1	0·5525	$-5{\cdot}36\times10^{-3}$	$6{\cdot}93\times10^{-4}$	0·2175

while for the second mode we obtain

	ϕ_0	ϕ_1	ϕ_3	ϕ_5	λ
(1)	−0·2788	1	—	—	2·260
(2)	−0·2745	1	0·01581	—	2·240
(3)	−0·2770	1	0·01571	$-1{\cdot}99\times10^{-3}$	2·241

The correct values of λ, obtained by the method of receptances, are 0·2167 and 2·238 respectively. Here again, the solution converges rapidly.

For the higher symmetrical modes, the effect of the added spring-mass system is small, and initial approximations may be obtained by assuming the co-ordinate $\phi_r = 1$, and neglecting the other co-ordinates. In the case of the skew-symmetrical modes, there is a node at the mid-point of the beam, and the frequencies and modes of the system are simply those of the beam alone.

In this problem the solution is complicated by the fact that the coupling term b_{01} is not small. For this reason it was necessary to include both ϕ_0 and ϕ_1 to obtain a reasonable first approximation. The theoretical solution may clearly be extended to a beam with several spring-supported masses. However, the numerical solution may be tedious owing to large coupling terms.

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The Performance of Ram-Jet Combustion Systems

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THE data presented in Dr. L. A. Povinelli's note published in the September 1960 JOURNAL (p. 564) makes an interesting addition to the now extensive literature on the characteristics of practical ram-jet combustion systems, but it is doubtful whether it provides anything new in the way of "specific subjects of interest for academicians."

The gap between the results of so-called "fundamental" research on combustion and the testing and development of practical combustion systems is still wide. This is because practical systems tend to have their own characteristics affecting the relationship between, and relative importance of, chemical processes, flow patterns, and heat and mass transfer. In isolation each of these subjects is complex. In association they are often baffling.

Dr. Povinelli's note describes and discusses some of the characteristics of particular systems. Many of his attempts at drawing generalised conclusions must be interpreted with reserve. It is hoped that the following comments, by way of specific criticism, may lead to deeper analysis and increased usefulness of the data given.

1. The fuel-air mixture temperature is known, in

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general, to exert a significant, and in some instances controlling, influence on combustion processes. The level and variation of temperature of the mixture in the experiments is not stated, though it appears to be just above atmospheric, nor is its influence discussed.

2. The experiments were conducted in fairly small chambers. Scale effects would be expected to be important here, though no reference is made to this.

3. The length diameter ratio of 4 used in many of the experiments is unacceptably high for subsonic ramjets for helicopter application, the acceptable range being 1 to 2 owing to the stressing conditions encountered in rotation. There are no other practical subsonic applications.

4. The data given in Fig. 1 is in many respects unsatisfactory. The "scatter" of the points is large—roughly equal to the difference in height between the two curves for which points are given. It is difficult to have confidence in the supposed separation of the curves, and therefore in any derived conclusions in this connection. Such "scatter" is typical of tests on practical systems, and emphasises the complexity of the inter-relationships between the factors mentioned earlier and their liability to vary between successive tests.

5. Reference to the influence of the presence of an exhaust nozzle on stability limits through its influence on the nodal pattern of pressure oscillations appears highly speculative. It is tempting, but often dangerous, to look for a single controlling factor in combustion system behaviour. The real picture is invariably complicated.

Dr. Povinelli's Reply

1. The approach mixture temperature very definitely exerts an influence on the flame stability characteristics, and this has been rather thoroughly discussed in papers on flameholding appearing in the Proceedings of the International Combustion symposia over the past several years. The experimental work reported in Ref. 1 was performed at ambient temperature, as mentioned in the original report.

2. As pointed out by Dugger and Gerstein[2], the effect of turbulence of given scale and intensity decreases with an increase in the ratio of flameholder size (*i.e.* diameter) to turbulence scale. Moreover, Childs[3], on the basis of Scurlock's results[4], has concluded that for flameholders of the order of one inch there is no measurable effect of approach stream turbulence on the flameholding characteristics. This author has arrived at similar conclusions based on an investigation performed at the Politecnico di Torino. Further scaling studies would, quite naturally, be necessary and useful.

3. The specific fuel consumption values obtained with the bluff body geometry became progressively poorer as the length/diameter ratio was decreased (4·00 to 3·52) in comparison to those obtained with the recessed geometry. The length/diameter ratio points up the important consideration of area distribution and improved mixing in the stabilising area in order to create greater flame spreading.

4. The effect of combustion chamber material on the flameholding ability of such laboratory systems is to be expected since the choice of material may strongly influence the heat transfer characteristics. The experimental results reported are in accordance with previous observations. The curves under question were obtained under identical operating conditions (*i.e.* upstream turbulence, mixture temperature, ram-jet configuration).

5. The Rayleigh criterion appears to be a logical explanation for the phenomena in question, notwithstanding various coupled effects

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The Hydro Biplane, Windermere, 1912

HERBERT à. BRASSARD

WHILE STAYING at the "Old England Hotel" Bowness-on-Windermere at the end of May in 1912 one heard and then saw what was loosely called an aeroplane flying over the Lake. It is of some "historic" interest that the car which brought us to Bowness was a 45 h.p. six cylinder Napier San Giorgio model. The Hydro Biplane was garaged in rather untidy looking wooden sheds only some 100 yards from the Hotel and I have a clear recollection of the following facts. The photographs (Figs. 1 and 2) show it to be a single-engined pusher propeller biplane on one central float. It always flew at a height of about 100 yards at around 50-60 m.p.h. A little speedier "downhill." Taking off speed was about 50 m.p.h. The pilot (*see* Fig. 2), I never knew his name, flew two or three times a day in suitable weather garbed like the present day motor cyclist but without crash helmet. The locals said the biplane was under test for the Admiralty. A small board fastened to a nearby tree near the sheds advised the sale of postcards and that passenger flights could be arranged. Having been told by the pilot that I flew with him at my risk and then having paid him four golden sovereigns, I seated myself on what was in fact a kitchen chair. I held on to the rods on each side of me and wore a cap in reverse. The engine spluttered into noisy but rhythmical revolutions. I recollect we taxied for some 50 yards and took off smoothly and so flew around Lake Windermere for some twenty minutes in all. It was interesting and not frightening at all but I did shiver with the cold air rushing around me. I remember that, when banking, I felt not unpleasantly glued to the chair, a gyroscopic effect, and there was no inclination for me to slip sideways or lean to the opposite direction. One was, or seemed to be, at the same angle as the plane. Flying "downhill" to land on the Lake was thrillingly speedy and there was no need to lean backwards or tend to fall forwards. Touching down was remarkably smooth and without much splash. And so back to the sheds. I suppose this episode may be termed almost "historic."

From H. F. Cowley

THE PHOTOGRAPHS which accompany Mr. Brassard's note are of the Lakes Flying Company's hydroplane "Water Hen." Captain E. W. Wakefield, of Kendal, pioneered the flying that took place at Lake Windermere and, in 1911, formed the Lakes Flying Company in conjunction with friends. In 1911 A. V. Roe and Company

built, for Captain Wakefield, a Curtiss-type hydroplane which was called "Water Bird." After successful tests as a landplane at Brooklands the machine was sent to Lake Windermere, where, piloted by Mr. Stanley Adams, it began flying in November. "Water Bird" made many flights but at the end of March 1912 a gale blew down the shed which housed it and it was destroyed.

Encouraged by the success of this first machine, Mr. Wakefield had started work on a similar hydroplane and it was in course of construction when "Water Bird" was broken up.

The new machine was named "Water Hen" and was a two-seat pusher biplane, constructed of wood and having wings of unequal span. The lifting and control surfaces were fabric covered. Bamboo outriggers bore the tail unit and also the forward elevator which was operated, in conjunction with the tail elevator, by a bamboo pole from the control column in a similar way to that used by S. F. Cody in his biplanes. Ailerons were fitted to the upper wing only. At first, a single central three-step float, 12 ft. long and six ft. wide, supported the machine on the water. This float was built by Messrs. Borwick and Sons, of Bowness, and had a flexible suspension system of rubber cord. The sides were of duralumin and the bottom was made of aluminium, the top being covered with Willesden canvas. Cylindrical air-sacks with spring board protectors, or stiffeners, steadied the hydroplane while taxying and they were known locally as "Wakefield Sausages." The power was provided by a 50 h.p. Gnome engine which drove a two-bladed propeller. Water spray, thrown up by the central float, caused damage to the propeller and it was found that the fabric covering was not sufficient to prevent the blades from splintering. Later on, the extremities of the propeller blades were covered with brass to protect them from the spray.

One of the first propellers to be used on "Water Hen" is still in existence and was carved from solid mahogany by Messrs. Borwick and Sons. Mr. A. I. Borwick tells an amusing story of how he came to lose the top joints from two of his fingers after swinging the propeller. Climbing down, after having successfully started the engine, he felt twinges of pain in one hand. "The tips of two fingers had been whipped off by the revolving propeller," he says, "and they flew off into the air and into the lake, and no one has seen them since."

"Water Hen" was tested by Stanley Adams on 30th April 1912 and, after minor adjustments, started its long career of passenger-carrying and pilot-training over Lake Windermere. Records show that, during the next seven months, this machine made some two hundred and fifty flights and carried over 100 passengers.

In August 1912 wireless messages were sent out from "Water Hen" to the lakeside and the Admiralty were very interested in these experiments.

Pilot-training was done in addition to the carrying of passengers for joy flights and it was on "Water Hen" that Lieut. J. F. A. Trotter obtained the Royal Aero Club Aviator's Certificate No. 360 on 12th November 1912.

From time to time the machine was modified. Different designs of central float were adopted and, with the seating arrangements adjusted, a nacelle to accommodate the two occupants was fitted. Later still, the central float was abandoned and twin floats were substituted, which allowed the wing-tip floats to be dispensed with.

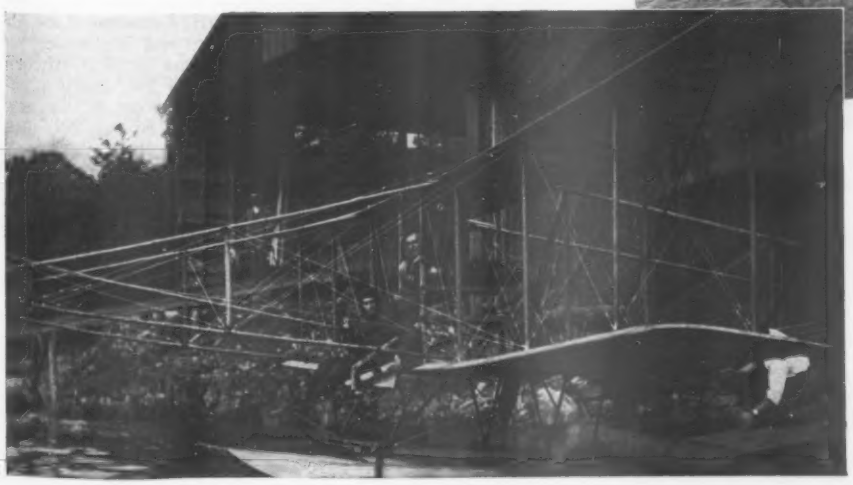
The Lakes Flying Co. was taken over, in 1914, by the Northern Aircraft Company, but "Water Hen" continued flying until 1916 during which time it did yeoman service and helped to train numerous pilots.

The instructors included Stanley Adams, Lieut. Trotter, Ronald Kemp, J. Lankester Parker, Rowland Ding and Cecil Pashley.

Note.—In those early days various terms were used to describe aeroplanes which took off from, or alighted on, water, including "Hydroplane," "Hydromonoplane," "Hydro-biplane," "Hydro-aeroplane" and "Waterplane." In July 1913, at the suggestion of the First Lord of the Admiralty, Mr. Winston Churchill, the term "Seaplane" came to be used universally.

Specification of "Water Hen"

Built by—Lakes Flying Co., Lake Windermere.
Description—Two-seat pusher hydro-biplane.
Construction—Wood, with fabric covering to the surfaces.
Engine—50 h.p. Gnome.
Dimensions—Span, 42 ft.; length, 36½ ft.; wing area, 365 sq. ft.
Weight—Empty, 780 lb.; loaded, 1,130 lb.
Performance—Max. speed, 45 m.p.h.; landing, 33 m.p.h.; ceiling, 800 ft.



Above:
FIGURE 1. The Hydro Biplane being launched on Lake Windermere.

Left:
FIGURE 2. The Hydro Biplane with its pilot (*left*) and a passenger.

Note: The photographs are from postcards bought in 1912 by Mr. Brassard and taken by Herbert and Sons, Windermere.

The second Branch to be formed in the 1930*s was Southampton*

IT IS now almost automatic on this page to record that the then Secretary of the Society, Captain J. Laurence Pritchard, attended a meeting—on this occasion one held at University College, Southampton, in April 1935, to discuss the formation of a Southampton Branch of the Royal Aeronautical Society.

The Secretary's attendance must have been effective, for the inaugural meeting of the Southampton Branch was held at the University College in September under the chairmanship of Wing Commander T. R. Cave-Browne-Cave, with Mr. S. Scott Hall as Honorary Secretary. One of the Committee was Mr. B. S. Shenstone, now a Director and Chief Engineer of B.E.A.

The inaugural lecture was given in October by the President of the Society, Lt.-Col. J. T. C. Moore-Brabazon, the title of his address being, "Whither Aviation?" It would be fascinating to read what Lord Brabazon had to say, but does a copy of this paper exist?

By December of 1935 the Branch membership had grown to 100, and things being on the up and up, it was decided that the time had come to ask Professor T. D. Hill to give a lecture on "The Stratosphere."

During 1937 the Branch held a joint meeting with the Institution of Production Engineers (Southern Section) and established closer collaboration with the University College of Southampton, and Air Service Training Limited of Hamble.

In their lecture programme for 1937 the Committee recommended that a paper should be read on "Air Raid Precautions for Factories" and in view of the subsequent bombing in the Southampton area with the effect on the Spitfire programme at the Vickers plant, the significance of that paper could only be appreciated during the war years, particularly at the time of the Battle of Britain.

In view of the rearmament programme and the activities in the southern area, the Committee met for the last time in June 1939, and Branch activities were suspended until April 1945, when an extraordinary general meeting, which was well attended, was held at the University College to re-form the Branch. Wing Commander Jenkins, on the original Committee as a Flight Lieutenant, became Chairman, and Mr. T. Tanner was elected as Honorary Secretary.

The first post-war year of activity consisted of lectures, and contacting as many of the former members as possible; in that year 171 members were entered in the Branch records.

1946 showed an increasing interest in the Society's activities and closer relationship was developed with the Isle of Wight, Portsmouth and Reading Branches. The changing pattern in the Industry was indicated by the lecture on the 19th December 1945 by W. F. Hilton, entitled "Compressibility Effects at Transonic Speeds." And for Southampton there is nostalgia in recalling a contemporary lecture by Mr. Arthur Gouge (now Sir Arthur) on "Flying Boats—Their Development and Possibilities."

In the sessions 1948 and 1949, the Committee noted with regret that there was a falling-off in the attendance, and Wing Commander Cave-Browne-Cave, on being re-elected Chairman of the Branch, urged the Committee to make every endeavour to further recruitment.

In March 1950 the late Professor G. T. R. Hill gave his memorable lecture entitled "One Thousand Miles Per Hour," which, at that time, seemed a little fantastic.

Professor E. J. Richards became Chairman in 1951, and his encouraging direction has guided the Branch through the difficult waters of post-war years. He has held this post off-and-on until the present day, and he has become synonymous with all matters relating to the Southampton Branch of the Royal Aeronautical Society. Previous to this Mr. H. C. Smith took over as Secretary from Mr. T. Tanner, who had served the Branch so well for fourteen years.

In 1952 the Committee decided to hold an annual R. J. Mitchell Memorial Lecture to commemorate the work of the famous designer who had made such a great contribution to our war effort, particularly in the design and development of the immortal Spitfire. The Inaugural Mitchell Memorial Lecture was the outstanding event of the year 1954 and was given by Mr. J. Smith, who had worked with Mitchell for many years. This was closely followed by the Second Memorial Lecture given by Mr. Peter Masefield. Both lectures were well attended, and set the standard for these important occasions.

1955 saw a greater interest in the presentation of papers, and in that year, four awards were made for papers read before the Branch. The Third Mitchell Memorial Lecture was given by Mr. Lovesey in 1956, attended by Mr. N. E. Rowe, President of the Society. The Chairman of the Branch at that time was Mr. H. C. Smith (the previous Chairman being Mr. D. B. Smith, not to be confused with the lecturer, Mr. J. Smith), and the Honorary Secretary was Mr. G. Hudson.

The 1956 and Fourth Mitchell Memorial Lecture was given by Professor Richards (who must have resigned, or been re-elected Chairman); the Secretaryship passed to Mr. S. Stapleton, who had a difficult time in that the Branch became concerned with the dropping off of membership and attendances at lectures, which was attributed to the recession in the Aircraft Industry in the Southampton area.

The Fifth Mitchell Memorial Lecture was given by Dr. S. G. Hooker, and another notable event of that year was the award to Professor Richards of the Order of the British Empire for "distinguished services in the field of engineering."

In the following year, continued recession in the Aircraft Industry in the Southampton area caused a heavy drop in the Branch membership, but in spite of this the Sixth Mitchell Memorial Lecture, given by Mr. Brennan, attracted the largest attendance of members for the year. F/Lt. V. Becker, of the Southampton University Air Squadron, acted in the capacity of Honorary Secretary until the present Secretary, Mr. F. C. Kirkpatrick, took over the duties in January 1959. Mr. D. B. Smith, in that year, resigned from the chair (he took over from Mr. H. C. Smith in 1957), and Professor Richards was again elected Chairman. The Branch has now become stabilised with a regular attendance at lectures of between 60 and 70 people, considerable support being given by the students of the Engineering Department of the University of Southampton. The Branch draws its membership from Vickers-Armstrongs, Folland Aircraft, Saunders Roe, Air Service Training and, as noted, from the University.

In 1959, Mr. D. Keith-Lucas gave the Seventh Mitchell Memorial Lecture and this proved to be the largest attended meeting of the year. With the opening of the new College of Air Training at Hamble, the Branch recruited to the Committee a member of the College staff; it is hoped to extend recruitment, and every endeavour is being made to establish closer relationship with the College.

The Branch asks that their debt to the University of Southampton be recorded for the help and facilities which have been most generously given since its formation in 1935.

It is an historical nicety that the Eighth Mitchell Memorial Lecture for the Session 1960/61 is to be given by Mr. B. S. Shenstone, a member of the Committee in the early formative years of the Branch. The title of the lecture is "Man Powered Aircraft." Admirable, even if it has no connection with the "Stratosphere" or "1,000 m.p.h."

Perhaps the discourse may be on the "Man-Powered Flying Boat," which will then answer Lord Brabazon's question. It certainly shows the wide interests of the Southampton Branch.—G.W.-W.

Graduates' and Students' Section

Letter from America—Part I

This letter was written by the Director of Student Activities of the American Institute of the Aerospace Sciences following an invitation from the Chairman of the Graduates' and Students' Section, so that members could learn more of the history, mode of operation and current activities of this "sister" organisation, and to further collaboration between the Groups.

The name of the man who first suggested the possibility of an Institute of the Aeronautical (as it then was) Sciences Student Programme is, unfortunately, not recorded. Presumably he was Lester D. Gardner, who, with Dr. Jerome C. Hunsaker, founded the Institute in 1932. At any rate, it is recorded that a grade of Student Member was announced early in 1935 and by Christmas of that year four Branches were in operation—at the Universities of Detroit (Michigan) and Cincinnati (Ohio), the University of Minnesota and at the Carnegie Institute of Technology in Pittsburg, Pennsylvania. These Branches had a combined membership of some 200 students.

In the Spring of 1940, Major Gardner, with that year's President, Major (now General) James H. Doolittle, made a series of visits to Branches, which at that time numbered 31. In the Spring of 1960, Branches were formed at Hartford University, in Connecticut, and at Drexel Institute of Technology (Philadelphia, Pennsylvania), to bring the total of Student Branches at the time of writing to 80, representing a little over 4,500 individual Student Members.

In 1954, the Student Programme received a grant from Glenn L. Martin of $500,000 to expand existing programmes and develop others in the light of the rapid growth of the industry and the profession. The Minta Martin Aeronautical Student Fund of the I.A.S. (named after Glenn Martin's mother) thus became the economic foundation on which the programme is built.

This brief skirmish into essential history over, it is perhaps appropriate to state the purpose of having a Student Programme at all. In keeping with the traditions of most societies the Student Programme has a code—a motto—which, to ensure full comprehension, is in English, as follows:—

"For the purpose of the advancement of the Aerospace Arts and Sciences, the encouraging of professional consciousness and fellowship and the broadening of professional relationships among Members."

The Student Programme works to translate that statement into a series of yearly activities in which the majority of Student Members will, of their own accord, want to participate; as with most mottoes, merely telling members achieves little. Showing them is better. Stimulating them to show themselves is best of all.

The core of the I.A.S. Student Programme lies within each Branch. Members elect their own officers, plan and run their own meetings (averaging two a month). Each Branch has a Faculty Adviser, invariably a member of the Aeronautical Department of the School, and a graded member of the Institute. His influence on a Branch is never under-estimated. Working with impressionable young men, as he and we are, his active support of a Branch does a great deal towards building up and sustaining interest among the students. Usually the Faculty Adviser is an extremely busy man without the additional responsibility of a Student Branch. It is certainly a testimonial to his own "professional consciousness" that in the past twenty-five years only a handful of Branches have been dissolved, and of these few, only a couple at the specific request of the Faculty. By exercising mature leadership, by showing interest in and showing up at virtually all Branch activities, the number of students these men have inspired to be leaders themselves is without doubt one of the most worthwhile dividends of the entire Programme.

Individual Branch meetings are varied. We encourage the scheduling of as many speakers as possible during the school year. A technical talk—as opposed to a technical film, a frequent alternative—is far better, as it brings student engineers together with professional engineers, gives them an opportunity for informal conversation as well as the chance to hear, first-hand, a specialist talk about his speciality. To assist Branches in obtaining speakers, Corporate Members of the Institute are canvassed every year to provide the names of the members of their engineering staffs who will lecture to Branches, as well as the areas of the country they are prepared to visit. This information is compiled and a Speakers List issued each year to all Branches. Corporate Members' response is good. The 1960 list, for example, names twenty-four companies to contact, and covers virtually all the major fields of aeronautics, including such (relatively) new engineering responsibilities as Computer Programming and Human Factors.

Branches do an encouraging job on their own of creating methods of communicating technical information. Several examples come to mind: a round-table symposium composed of students and faculty from Engineering and other Departments of a school in which there is a related interest (*i.e.* a discussion of survival in space at one Branch effectively tapped the opinions of those in physics, chemistry, sociology and psychology); a meeting devoted to comments from senior students who worked in industry during the long summer vacation; another meeting at which faculty members of the Engineering Department speak on the career opportunities and the merits of their respective specialities; joint meetings with other Branches in the area, as well as with local senior Sections of the Institute.

Branches also organise field trips, often travelling long distances to visit industry and government research and/or production facilities. They participate in Science Fairs, sponsored by local communities for the benefit of school children, and in "Open Houses" held by their own universities. One Branch is currently building a full-scale replica of the Wright Brothers' aircraft and plans to test fly it this Autumn. Another built a Ground Effect Machine and exhibited it at an "Open House" where it "flew some 10 to 12 in. off the ground for more than 10,000 visitors" who attended.

ANTHONY W. HARRIS.

(To be continued)

Future Visits

Visits have been arranged to: Lotus Components, Cheshunt, Herts, on Saturday morning, 19th November 1960, and to Associated-Rediffusion Television Studios at Wembley Park, on Tuesday evening, 6th December 1960. Members wishing to attend either or both of these visits should apply to the Hon. Visits Secretary, N. R. Craddock, 5 Oxleay Road, Harrow, Middlesex (Tel.: Pinner 3633) *at least a fortnight beforehand.*

Annual Dance: Friday 25th November

The dance this year will be the last one to be held in the Library of the Society at 4 Hamilton Place, W.1, as future functions will be held in the new Lecture Theatre. Dancing will be, as usual, to the music of Hugh McCamley, and double tickets, which include free drink and refreshments, will again cost only 14s.

Your tickets may be obtained from your nearest Committee member or representative, or write to Mr. W. G. Wilson, 107 Queen's Gate, S.W.7. *Cheques and Postal Orders should be made payable to Graduates' and Students' Section, Royal Aeronautical Society.*

November Lecture

The first lecture of the Autumn Session, on Wednesday 23rd November, will be given at 7.30 p.m. in the Library, by M. O. Robins (University College), on the subject: "The British Space Programme." Coffee and biscuits will be served at 7.0 p.m.

THE LIBRARY

Reviews

THE AERODYNAMICS OF POWERED FLIGHT. *R. L. Carroll. John Wiley, New York (Chapman and Hall, London). 1960. 275 pp. Diagrams. 68s.*

The author states in the Preface that "This book is intended to serve as an introduction to the aerodynamics of powered flight . . . ." at undergraduate level, and arises out of the author's lectures to the U.S. Navy test-pilot school at Maryland. In passing judgment on this book it might be noted that this course covers a period of eight months, is extremely concentrated, and the ages of the students range from the early twenties to about forty. It is therefore not surprising that the number of topics included is very large and that none are treated beyond a certain elementary stage. In general the treatment is non-mathematical and the derivations of formulae are either not given or are based on physical arguments only. Although this approach is moderately successful in the early chapters, dealing with lift, drag and the relationship between velocity and pressure, the space allocated for boundary layers, and subsonic, transonic and supersonic aerodynamics and other topics is so small that it is not surprising many terms are poorly and in some cases inaccurately defined and the uninformed reader could be easily misled. The title of the book is also somewhat misleading even if most of the author's remarks do relate to fixed wing aircraft. It does not deal with the problem of powered lift.

The first chapter introduces the mechanics of flight simply and effectively using a simple rocket as the basic example. Lift, downwash and propulsive efficiency are thereby obtained in a clear manner, which would have some appeal to students being introduced to aerodynamics for the first time. Then follows chapters on pressure distribution, lift and drag. Basic aerofoil theory is not introduced and the concept of circulation is derived from the consideration of a rotating cylinder in a viscous flow. The difference between results for two-dimensional and finite span aerofoils is not made clear and is most noticeable in the remarks relating to lift-drag ratio and aerodynamic centre.

The chapter on boundary layer effects is totally inadequate. We are told on page 69 that, ". . . . In the absence of turbulent flow there is a fairly abrupt transition from the boundary layer to the freestream flow. This sharp energy gradient . . . ." Such a statement, together with Figs. 5.1.1 and 5.1.3, is utter nonsense. Further no distinction is made between the effects of leading and trailing edge stall so that the results quoted in successive figures give apparently quite conflicting results.

The chapter on compressibility effects promises well but falters badly after deriving a $C_p(M)$ relationship for a thin aerofoil based on one-dimensional flow considerations. The author even goes so far as to compare his so-called new result with those of Glauert and von Kármán as well as with experiment. One might excuse him for this on the grounds of simplicity in derivation except that he goes on to use a similar formulae for C_L, C_m and C_D! Similarly it is difficult to overlook the statement in Chapter 7 that the terminal velocity of an aircraft increases with either an increase in C_D or an increase in atmospheric pressure.

The essentials of the flow associated with shock waves and expansion waves are given in Chapters 8 and 9 and their applications to supersonic thin aerofoil theory are given in Chapter 10. The author omits to mention that the skin friction coefficient depends, among other factors, on both Reynolds number and Mach number. In addition no mention is made of the effects of rate of heat transfer. The description of the transonic drag and transonic and supersonic area rules are of interest but are too vague to be of any real use.

After dealing with the elementary theory of the propeller the author proceeds to a short account on aeroplane performance. In this chapter the treatment is spoilt by lack of attention to detail. For instance, the speed range below the minimum drag speed is given as that associated with speed instability without specifying the thrust-speed characteristic. This gives a false impression of the problem, as indeed do the various figures in this chapter, which give results for varying speed without specification of the altitude and without stating whether true or equivalent air speed is used.

The attempt to avoid a mathematical approach makes the chapter on stability of little use to the more serious student of aerodynamics. Trim and stability are muddled and the instability resulting from the rapid variation of parameters with Mach numbers is ignored.

The book closes with a short chapter on experimental methods, including radio telemetry, while Appendices give respectively wind tunnel test data on an aerofoil (which is hardly typical since the maximum lift/drag ratio is 8·5 at low speeds) and pilot-induced oscillations.

It is not without some interest to note from the Index the wide diversity of subjects listed—black box, Bureau of Aeronautics, feedback, high-energy fuels, M and W wings, mission profile, Navy Test Pilot School, noise suppression, radio telemetry and reversed command—to mention just a few.

The book is stacked with elementary examples of which half are given with answers. The format is good but the diagrams are spoilt by poor titling. With corrections the book could be used to introduce aerodynamics students who do not require a specialist knowledge of the subject.—J. J. SPILLMAN and G. M. LILLEY.

BASIC ELECTRONICS. *6 parts. Van Valkenburgh, Nooger and Neville. Technical Press, New York, 1959. 50-60 pp. each part. Illustrated. 12s. 6d. per part. 66s. set of six.*

These six small volumes continue a similar five-part Training Course in Basic Electricity (reviewed in the July 1959 JOURNAL, p. 423), with two more volumes to come on Synchros and Servomechanisms, and three covering Basic Radar. They provide useful groundwork, for technicians rather than engineers, or, in the words of the authors, aim to "turn out men capable of operating, maintaining, and carrying out routine repairs, (but) not men capable of inventing or improving it."

That is a modest enough claim, and the books more than adequately fulfil the promise, within the compass of a basic course of instruction.

Treatment is thus necessarily descriptive and explanatory; presentation and style are easy and readable; or, from another aspect, objective and practical. In considering more abstract principles, the text is supported by a wealth of good diagrammatic illustration and sufficient numerical discussion, both in the text and diagrams.

Main emphasis is on basic radio circuitry—amplifiers, oscillators, transmitters, receivers, amplitude and frequency modulation, and transistors—but non-sinusoidal waveforms and multivibrators are also dealt with at fair length. Indeed, for an "elementary" text, one is impressed with the range of subject matter and the thorough way in which it is discussed.

Each main section ends with an aptly illustrated "Subject Review," followed by a few pages in the way of a "General Review" at the end of each volume.

If criticism is needed, one may question whether, here and there, the authors do not fall short of the ideal of "extreme lucidity" which they set themselves. Thus, definitions occasionally tend to be left "hanging" through too ready a use of the words, "This is called . . . . .". One has in mind definitions such as: "the process of increasing voltage is called amplification," or, "it (a mains supply) may, however, in practice contain components at much higher frequencies. *This* is called interference" (reviewer's italics). Possibly a few elliptical passages result from adapting for English readers.

In teaching the fundamentals of amplification, a basic text should make clear at an early stage that there is a little more to it than mere voltage step-up.

An elementary treatment of the tuned-anode/tuned-grid oscillator with the aid of simple vectors is interesting, especially as sinusoidal oscillators are still with us in transistorised forms. The authors bring out the essential practical point, that the anode and grid circuits must be distuned to present inductive reactance. However, since this discussion of basic phase relationships in oscillators *is* included, one would have welcomed a few earlier fundamentals, *e.g.*, whether oscillation can be maintained if a *component* of V_g is anti-phased to V_a (not necessarily the whole of the V_g vector); is the basic assumption that I_a is in-phase with V_g always true, *e.g.*, in the case of a triode with reactive anode load. The reader should perhaps be told that "vectorising" becomes complicated—with whole networks of reactances—at high frequencies, or that (within the limits of the possible) oscillation may occur at frequencies which *fulfil* the vector relationships necessary to maintain it.

This one example will serve to show some of the stimulating topics considered and the impetus they should give to further inquiries among readers who may want to know more.

The text as a whole can be well recommended as providing a solid basis for more recondite studies.—H. REES.

ADVANCES IN ASTRONAUTICAL SCIENCES. *Plenum Press, New York (Chapman and Hall, London), 1958. Published by the American Astronautical Society in four volumes. Illustrated. 80s. each Vol.:—Vol. 1, 177 pp.: Proceedings of A.A.S. Third Annual Meeting, December 1956. Contains 11 papers on satellite applications, electronics, space medicine, cosmic ray research, meteors, radio tracking, and dynamics; authors include C. S. Draper, H. Wexler and R. P. Haviland. Vol. 2, 450 pp.: Proceedings of A.A.S. Fourth Annual Meeting, December 1957. Contains 35 papers, plus 10 abstracts, on propulsion, dynamics, propellants, space medicine, aerodynamics, guidance, radio and telemetry, plasmas, and so on; authors include H. O. Strughold, H. J. Schaefer, G. W. Hoover, N. S. Davis and N. V. Petersen. Vol. 3, 540 pp.: Proceedings of A.A.S. First Western Regional Meeting, August 1958. Contains 44 papers, plus 7 abstracts, on a very wide range of subjects similar to Vol. 2. Authors include L. E. Root, A. J. Eggers, D. G. Simons. Vol. 4, 460 pp.: Proceedings of A.A.S. Fifth Annual Meeting, November 1958. Contains 36 papers, plus 4 abstracts, again covering a field similar to Vols. 2 and 3. Papers are divided into 6 sections. Upper Atmosphere Research and Re-entry: Space Vehicle Design: Guidance and Instrumentation: Satellite Mechanics and Space Exploration: Rockets and Satellites: Man's Space Environment. Authors include H. L. Dryden, J. A. van Allen, S. F. Singer, W. H. Pickering.*

In the early nineteen-fifties, a state of affairs existed in the United States which may seem surprising when viewed with post-sputnik hindsight, though it was not at all surprising at the time. In those days, bodies like our own Royal Aeronautical Society and the Institute of Aeronautical Sciences across the Atlantic only very occasionally concerned themselves with space-flight; when they did, it was usually to recognise, with a certain gracious air of condescension, that it was a possibility of the not-too-immediate future. Even the body which had been founded in 1930 as the American Interplanetary Society had, some years previously, changed its name to the American Rocket Society, and was energetically cultivating scientific respectability by devoting nearly all its efforts to rocket engineering, missile technology, and even ram-jets: at one time, it even called its Journal *Jet Propulsion*.

However, there were a good many Americans, including a number of competent engineers and scientists, who believed that there were much more important applications for the rocket than guided weapons and ballistic missiles, that engineering was not the only technology involved, and indeed that some of the most important considerations were not technological at all. A lot of these people joined the British Interplanetary Society, which had obstinately kept its colours nailed to the mast ever since its foundation in 1933; some of them founded the American Astronautical Society, which was largely modelled on the B.I.S., and which has continued to exist in parallel with the rival A.R.S. ever since.

In many ways, it is perhaps to be regretted that the two American societies have not followed the fashion of the times and merged, because the astronautical activities of the larger American Rocket Society have left nothing to be desired for at least the past five years, and inevitably there has been a great deal of duplication of effort.

Nevertheless, the A.A.S. has done some very good work, as these four volumes testify. The papers included in them are of course mostly on the short side, but their general technical level is high and their content of original thought was quite considerable at the time of first presentation. The books deserve a place on the shelves of any comprehensive library of rocketry and astronautics. However, in view of their price, and the present date, it is only fair to observe that the purchaser will now find relatively little in them which is not also available elsewhere. As a single set of references (themselves with a good bibliography), he could still do much worse than consult these collected Proceedings.—A. V. CLEAVER.

THE SMIRNOFF STORY. *Anne Robertson Coupar. Jarrolds, London, 1960. 271 pp. 25s.*

Ivan Smirnoff was one of those buccaneering personalities of the early days of air transport who played such an important part in laying the foundations of today's great airline industry. A Russian by birth and a pilot in the Czar's service during the First World War, Smirnoff went to Holland—by way of England and Belgium—after the Russian Revolution and it was as one of K.L.M.'s captains that he made his name and became a world figure in commercial aviation between the wars.

Smirnoff started as a civil pilot with the Belgian airline SNETA—SABENA'S predecessor—in 1921 but transferred to Holland's K.L.M. in May 1922 about a year after that company started operating its own services. (K.L.M. relied initially on chartered aircraft and crews.) In the years that followed, Smirnoff became a familiar figure on K.L.M.'s routes, first in Europe and, from 1928, on the long haul down to the Dutch East Indies (now Indonesia) as well. He had his share of adventures—including a forced landing on the Goodwin Sands and, after the Second

World War had spread to the Far East, being shot down by Japanese fighters on a remote beach in Northern Australia. During this period he was flying first in Europe, then in the East Indies and, later still, in Australia. He finished the War on K.L.M.'s Bristol-Lisbon run. With the return of peace, Smirnoff was soon back on normal line flying and, by 1947, had accumulated 27,000 hours which was perhaps the highest total accumulated by any pilot up to that time. After two years at a desk job he finally retired from K.L.M. in 1949.

Smirnoff went to live in Mallorca in 1950 and it was fortunate that, some years later, he met Mrs. Coupar there. She is an experienced journalist who happened to be on holiday in the same village. Becoming interested in his story she, with his assistance, undertook the task of writing his biography. This was clearly no easy task: firstly, because the author knows nothing about aviation and, secondly, because Smirnoff was not the sort of man who kept any complete record of his activities. Presumably his flying log books were available and have been consulted but one gets the impression that the book leans heavily on Smirnoff's own verbal reminiscences to Mrs. Coupar and that the author has had only a limited amount of help from private papers, from K.L.M. and from a few other friends in Holland. Smirnoff's memories seem, in fact, to have been the main source of material for this book which was finally completed by Mrs. Coupar and published quite recently following Smirnoff's death from cancer in 1956. Mrs. Coupar is generously giving the proceeds from the book towards cancer research.—PETER W. BROOKS.

THE EXPLORATION OF SPACE. *R. Jastrow (Editor). Macmillan, New York, 1960. 160 pp. Illustrated. 38s. 6d.*

This book contains thirteen papers and the reports of three round-table discussions on developments in the space sciences which have been published by the American National Academy of Sciences in August and November 1959 issues of the *Journal of Geophysical Research.*

In August and September 1958 three small atom bombs were detonated beyond the atmosphere to study the effect of trapping electrons in the earth's magnetic field. This Argus experiment, as it was called, produced a shell of electrons around the Earth from which further study of the Van Allen Radiation Belts, the greatest discovery of the International Geophysical Year, could be made. Papers describing both these studies are given in this book, along with papers on such subjects as the atmospheres of the planets, the origins of meteorites, relationships between solar corona and cosmic ray events, plasma and magnetic fields in the solar system, and rocket astronomy.

The suitability of American space vehicles, both existing and under development, for both lunar and planetary exploration, is the subject of a paper by H. E. Newell of N.A.S.A. entitled Capabilities for Space Research, and provides a comprehensive survey of important developments in space vehicles to the beginning of this year.

It is a welcome change to find that the background behind some particular experiment is described, and it is nice to be able to follow the logical steps which led to a particular method of approach being employed to achieve certain results. All too often the underlying reasons for this are omitted from papers and there is a common tendency for papers to evaluate the results of an experiment and form conclusions without giving the reader a fair chance to understand why the particular experiment was undertaken in that way, and, in some cases, even why it was ever undertaken at all.

All the papers are written in a nice easy style and the text is remarkably free from mathematical formulae, only the necessary minimum having been included. Considerable attention has been paid to the illustrations and the clarity of the graphs is to be recommended.—M. D. HULL.

HEAT EXCHANGERS—APPLICATIONS TO GAS TURBINES. *W. Hryniszak. Butterworths Scientific Publications, London, 1958. 343 pp. Diagrams. 63s.*

This books deals primarily with the thermodynamic and mechanical design of the different types of heat exchanger used in gas turbine units, and its effect on the suitability, performance and economy of the plant.

Development trends, purposes, types, parts, drawbacks and flow characteristics of heat exchangers and the recuperative and regenerative methods of heat exchange are discussed in the first three chapters. Then follow two chapters which deal with the effects of three heat exchanger internal efficiencies based on temperature, pressure and mass flow respectively, on the performance of the gas turbine unit: optimum conditions referred to minimum specific air consumption, minimum fuel consumption and minimum specific fuel consumption are analysed.

In Chapter 6, the internal efficiencies are related to the main properties of the heat exchanger matrix, and the use of extended surface explained. Both internal and external optimisation are discussed.

The next chapter is devoted to the more detailed design of recuperative heat exchangers. Heat transfer, flow resistance, matrix and header design for tubular matrix heat exchangers are discussed: plate matrices and the heat exchanger with a fluid agent are also covered.

The design problems, operation and sealing of regenerators with static and moving matrices are dealt with in Chapter 8: descriptions are included of three types of regenerators with a fluidised matrix.

In the next two chapters, an analysis is made of the part load performance of an air preheater, followed by the influence of the latter on the governing of the gas turbine unit, while its specific application to gas turbine units in vehicles is discussed in a later chapter.

The final chapters are concerned with the testing, materials, stressing and economics of heat exchangers and general design problems which affect the gas turbine unit.

The book is very well illustrated with 138 figures, which generally serve as an excellent complement to the text: there are exceptions, however, such as Figure 21, in which the example given is different from that in the text.

There are some useful data contained in the 45 tables, although the values of the adiabatic coefficients in Tables IV and V are quoted to an incorrect order of magnitude, and omission of units in the lower half of Table XII tends to give a false impression of the thermal conductivity of the insulating materials.

Inevitably, with a book of this nature, there are numerous subscripts in the 335 equations, and as many of these equations have been quoted without proof, the reader may have difficulty in recognising any errors which may occur, such as in the section on porous matrices on page 178. Particular mention must be made of the valuable lists of references, including in some cases the patent situation, which follow most chapters.

The reviewer would have preferred to have seen the term " moment of inertia " used instead of " momentum " in the section on forces and stresses.

To sum up, it is felt that this book should be a useful contribution to the improvement of heat exchangers, and should succeed in its aim of supporting research, project and development engineers in this work.—K.W.N.

DESIGN FOR FLIGHT. (The Kurt Tank story.) *Heinz Conradis. Macdonald, London, 1960. 246 pp. Illustrated. 21s.*

Tank was technical chief at Focke-Wulf from 1931-1945. This is the story of Tank's aeroplanes rather than of the designer himself. He was just old enough to serve in the First World War and then went to the Berlin Technical High School. He designed and built a glider for the famous Rhön meeting of 1923 and was persuaded to enter the reviving Aircraft Industry. With the firm of Rohrbach he went through the process of being told that his ideas were nearly all impracticable— monoplanes, metal construction, canopies for cockpits, dive bombing. He survived this and joined Focke-Wulf in 1931. Professor Focke was engrossed with his helicopters and Wulf had died in 1927. Tank had a free hand and was responsible for the many famous designs—responsible not only for their design but for much of the test flying. (One nostalgically remembers the old days when flying machines carried men, even if their designers seldom flew. Before that some designers actually piloted their own planes!).

The most famous were the Fw200 Condor, a four-engined transport for Lufthansa which flew in 1937. It made trans-Atlantic flights in 1938 and was ordered by foreign airlines. During the war 130 were built and used on anti-shipping raids. The Fw190 was the fighter which aroused so much admiration in Britain when the first captured one was flown at Farnborough in 1942. It was, at that time, the fastest fighter in Service and was very interesting with its electrically operated flaps and undercarriage (Tank had graduated in electrical engineering). Later models reached 470 m.p.h.

The story up to 1945 tells us very little about Tank the man, but is excellent material for the aircraft historian. A memorandum by Tank, printed in full, on the causes of the defeat of the Luftwaffe is interesting. He thought the Me109 and Fw190 superior to the Spitfire and Hurricane but of too small a range to win the Battle of Britain; the Luftwaffe had poor technical direction and were provided with bombers rather than fighters. He thought the American day bombing more damaging to German industry than the R.A.F. night bombing.

After the war, like other German scientists, he awaited invitations to work abroad. The Russians got in first and he went to Berlin to be interviewed by Col. Tokaev but their offers were too vague to tempt Tank. In 1947 he was offered a job in England with Handley Page but refused. He continued his design work in Germany but late in 1947 was ordered to come to England. He fled in disguise via Denmark, thence via London Airport in a British aeroplane(!) to Argentina where he was later joined by 60 members of his design team to work for the Argentine government. By the mid-fifties Tank's second totalitarian employer had been overthrown and he considered a return to Germany but a revival of the Aircraft Industry there still seemed remote, so Tank accepted the task of helping India create an industry and still works there.

Although the author worked with Tank for many years, he tells us almost nothing of the man. Tank was a good engineer, a good and lucky pilot, a hard worker. Perhaps he had no time to be anything else.—A. H. YATES.

Additions to the Library

Air Navigation. U.S. Department of the Air Force. U.S.G.P.O. Washington. H.M.S.O. 1959. Sectional pagination. Diagrams. 27s. Air Force Manual 51-40, it supersedes the same series number of 1954. The first of three volumes, deals with aerology, earth and its co-ordinates, basic instruments, dead reckoning, pressure differential techniques, radio, Loran and radar.

Analysis and Design of Feedback Control Systems. G. J. Thaler and R. G. Brown. McGraw-Hill, London. 1960. 648 pp. Diagrams. (Second edition of *Servomechanism Analysis*). 112s. 6d. The previous edition appeared in 1953 under the title *Servomechanism Analysis* and was reviewed in the September, 1953, JOURNAL (page 604), where it was highly recommended as a "lucid and well-arranged exposition of the theory of servomechanisms." The revision has obviously been a major one, including several new chapters, reflecting the considerable development of the subject which has taken place.

Atlas: The Story of a Missile. J. L. Chapman. Gollancz. London. 1960. 190 pp. Illustrated. 21s. *To be reviewed.*

Aviation Cartography. Walter W. Ristow. Card Division, U.S. Library of Congress, Washington 25 D.C. 1960. 245 pp. $1.75. It may be thought that there is but a small literature on the subject of aviation maps, but this work, with its subtitle "A Historico-Bibliographic Study of Aeronautical Charts" (in its second edition) has a bibliography of 774 items. The historical study which precedes this bibliography deals with conception and birth, 1888-1908, maps for powered and sustained flight, war maps, 1915-18, commercial navigation 1919-39, a period when maps were produced by the million (1940-48) and, finally, "A Dramatic Decade 1949-59," dealing with the developments of radar, modern navigation aids and the like. Unlike some bibliographies, this one draws its material from international sources.

Basic Synchros and Servomechanisms, Parts 1 and 2. Van Valkenburgh, Nooger and Neville. Technical Press, New York. 1960. Sectional pagination. 16s. each part. Covers a course of training developed by a firm of management consultants and "graphiological engineers" for the U.S. Navy. Aimed at U.S. Navy Training Schools the books are essentially fundamental and are profusely illustrated by easy-to-read diagrammatic illustrations.

Claude Grahame-White. Graham Wallace. Putnam, London. 1960. 256 pp. Illustrated. 35s. *To be reviewed.*

de Havilland's Fifty Years. de Havilland Aircraft Co. 1960. 27 pp. Illustrated. A supplement to the August *Gazette* giving a largely pictorial review of the company's history.

Famous Bombers of the Second World War, Second Series. William Green. Macdonald, London. 1960. 136 pp. Illustrated. 21s. The companion volume to the first series reviewed in the May, 1960, JOURNAL (page 308). It contains short histories, fully illustrated, of the Bristol Blenheim, the Mitsubishi Ki. 21 Type 97, the Armstrong Whitworth Whitley, the Handley Page Hampden, the Short Stirling, the Mitsubishi G4M Type 1, the Douglas A-20, the Focke-Wulf Fw 200C Condor, the Handley Page Halifax, the Heinkel He 117 Greif, the Boeing Superfortress, the Mitsubishi Ki. 67 Type 4 Hiryu and the Arado Ar 234B Blitz.

First Annual Report 1959, U.S. Federal Aviation Agency. U.S.G.P.O. Washington. 1960. 42 pp. (2s. 8d. from H.M.S.O.) A "man-in-the-street" report without technicality. A glossary of Federal Aviation terms is appended, giving the impression that F.A.A. has a language of its own, yet one finds such familiar items as Airport Traffic Control Tower, Radio Range, Turbojet and Turboprop.

Fundamentals of Rocket Propulsion. R. E. Wiech and R. F. Strauss. Reinhold, New York (Chapman and Hall, London). 1960. 135 pp. Illustrated. 44s. *To be reviewed.*

General Dynamics of Vibrations. Y. Rocard. Crosby Lockwood, London. 1960. 522 pp. Diagrams. 80s. *To be reviewed.*

Handbook of Supersonic Aerodynamics, Section 18, Volume 6. I. I. Glass and J. G. Hall. U.S.G.P.O., Washington (H.M.S.O.). 1959. 603 pp. Diagrams. $3.75. *To be reviewed.*

Helicopters. J. W. R. Taylor. Ian Allan, London. 1960. 63 pp. Illustrated. 2s. 6d. The Allan "abc" series is familiar to all readers and their sons and it is enough to announce that a new one is available. Five years have passed since this one's predecessor was published and in that time many rotors have performed many revolutions. The author includes such items as the Bell Air Scooter, the Vanguard Omniplane, one-man strap-on types and the rest. To find any type you have to know the manufacturer. The rest follows since the book is in abc order of maker and has no index.

Hero, The; Charles A. Lindbergh. The Man and the Legend. K. S. Davis. Longmans Green, London. 1960. 445 pp. Illustrated. 30s. *To be reviewed.*

International Missile and Spacecraft Guide. F. I. Ordway and R. C. Wakeford. McGraw-Hill, London. 1960. 411 pp. Illustrated. 194s. *To be reviewed.*

Magnetodynamics of Conducting Fluids, The. D. Bershader (*Editor*). Stanford University Press (Oxford University Press). 1960. 145 pp. Illustrated. 36s. Seven papers are contained in the proceedings of the Third Symposium on magnetohydrodynamics held in November 1958, at the Palo Alto research laboratory of the Lockheed Missiles and Space Division. In contrast to the wider scope of the two earlier symposia, this third meeting was characterised by concentration on the macrodynamics of conducting fluids. The discussions are not included.

Mathematical Tool-Kit for Engineers, A. H. A. Webb and D. G. Ashwell. (Second Edition.) Longmans Green, London. 1960. 116 pp. 15s. In this enlarged edition, new chapters have been added on complex numbers, the Laplace transform, the error function and some little-known extensions of Macaulay's method for discontinuous particular integrals. An index is a welcome addition to this useful handbook.

Mechanical Properties of Structural Materials at Low Temperatures. R. M. McClintock and H. P. Gibbons. U.S.G.P.O., Washington. 1960. 180 pp. (13s. 6d. from H.M.S.O.) Monograph No. 13 of the National Bureau of Standards, this contains eight pages of textual introduction; the remainder is a graphical presentation of the tensile strength, yield strength, tensile elongation and impact energy of about 200 materials, metallic and non-metallic, as functions of temperature between 4° and 300°K.

Proceedings of the I.U.T.A.M. Symposium on the Theory of Thin Elastic Shells. W. T. Koiter (*Editor*). North-Holland, Amsterdam. 1960. 496 pp. Illustrated. 55s. This contains 23 papers presented at the Symposium at Delft in August 1959, by invited experts from thirteen different countries. Two major topics in which research is currently active were emphasised, namely, the nonlinear theory and problems lacking axial symmetry. The two committees concerned were under the chairmanship of W. T. Koiter.

Proceedings of the Propellant Thermodynamics and Handling Conference. L. E. Bollinger and A. W. Lemmon (*Editors*). Engineering Experiment Station Special Report No. 12. Columbus, Ohio State University. 1960. 590 pp. $4. This book contains the proceedings of a specialist meeting of the American Rocket Society arising from the growing importance of high energy propellants in space flight and of especially hydrogen as a fuel. The meeting was held at the Ohio State University in July 1959, when some forty papers were presented, all by authors from the United States. Discussions are not included but all except two of the papers presented are included.

Proceedings, Xth International Astronautical Congress, London 1959. 2 Volumes. F. Hecht (*Editor*). Springer, Vienna. 1960. 504 and 441 pp. £19 11s. 6d. *To be reviewed.*

Properties of High-Temperature Ceramics and Cermets. U.S. Bureau of Standards. U.S.G.P.O. Washington. 1960. 45 pp. (1s. 10d. from H.M.S.O.) Monograph No. 6 in the N.B.S. Series, by S. M. Lang. Consists of an introduction, a description of the materials and procedure used and tabulation of results with discussion. 23 references are appended.

Radio Aids to Civil Aviation. R. F. Hansford (*Editor*). Heywood, London. 1960. 623 pp. Illustrated. 126s. *To be reviewed.*

Russian-English Dictionary. A. I. Smirnitsky. 951 pp. 50,000 words. 1959. **English-Russian Dictionary.** V. K. Müller. 1192 pp. 70,000 words. 1960. State Publishing, Moscow. (Collet, London). 31s. 6d. per volume. Remarkable productions at any price. The reader applied his usual yardstick and those words which he looked for in the English-Russian version were there. At the end is a list of proper names including such versions as Charlie (or Charley) and Jack. Another list is of geographical locations and yet a third of abbreviations.

Servomechanisms. P. L. Taylor. Longmans Green, London. 1960. 418 pp. Diagrams. 42s. *To be reviewed.*

Some of the Few. J. P. M. Reid. Macdonald, London. 1960. 60 pp. Illustrated. 5s. A series of illustrations of outstanding R.A.F. personalities by Cuthbert Orde, each accompanied by a short note listing their achievements. A royalty on each copy sold will be paid to the R.A.F. Benevolent Fund.

Standardization Activities in the United States. Sherman F. Booth. U.S.G.P.O. Washington. 1960. 210 pp. $1.75. Miscellaneous Publication 230 of the United States National Bureau of Standards, this is a descriptive directory of about 350 American bodies involved in standardisation. There are over 500 subjects listed in the index.

Symposium on Fatigue of Aircraft Structures. American Society for Testing Materials, Philadelphia. 1960. 138 pp. $4. Nine papers presented during sessions held in October 1959 at San Francisco. Subjects dealt with are quantitative and systematic attack on fatigue, development of fatigue loading spectra for testing aircraft components and structures, a method for sonic fatigue testing, low-cycle fatigue strength of pressurised components, fatigue testing of a large helicopter rotor blade, cold working joints subjected to fretting, predicting rate of fatigue-crack propagation, cycle ratio in cumulative damage and variable-amplitude fatigue tests of aluminium alloy. Each paper is accompanied by a discussion. Two other papers read at the symposium will appear at a later date.

Theory of Detonation. I. B. Zeldovich and A. S. Kompaneets. Academic Press, New York. 1960. 284 pp. Diagrams. $10. *To be reviewed.*

V.C's of the Air. J. F. Turner. Harrap, London. 1960. 187 pp. Illustrated. 11s. 6d. *To be reviewed.*

Warplanes of the World. J. W. R. Taylor. Ian Allan, London. 1960. 96 pp. Illustrated. 10s. 6d. Having got over the initial shock of "warplanes" (peaceplanes?) one realises that this is a sort of big brother of the familiar "abc" series. There are 238 entries and an index. The book is divided into two parts—major operational types and second-line aircraft—each of which is in alphabetical order of manufacturer. It is claimed in the introduction that 74 of the entries are completely new. That is to say they do not appear in the current abc's.

Welding Handbook, Part III. Special Processes and Cutting. 4th Edition. A. L. Phillips (*Editor*). American Welding Society (Cleaver-Hume, London). 1960. 510 pp. Illustrated. 72s. This is the third of five parts of the fourth edition of the Welding Handbook published by the American Welding Society. As well as covering established cutting and welding processes in detail, this edition contains new chapters on Welding of Plastics, Ultrasonic Welding, Welding by Cold Working and Adhesive Bonding of Metals.

Years of the Sky Kings, The. Arch Whitehouse. Macdonald, London. 1960. 334 pp. Illustrated. 25s. *To be reviewed.*

Reports

AERODYNAMICS

BOUNDARY LAYER *see also* INTERNAL FLOW
THERMO-AERODYNAMICS
WINGS AND AEROFOILS
THERMODYNAMICS

Pressure fluctuations in an incompressible turbulent boundary layer. G. M. Lilley. C.o.A. Report 133. *June* 1960.
In a recent pape by Lilley and Hodgson an approximate analysis is given of the pressure fluctuations on a rigid wall under a turbulent boundary layer. Here a more exact analysis is presented by using the method of generalised Fourier transforms.—(1.1.3.1).

Wind-tunnel tests on the use of distributed suction for maintaining laminar flow on a body of revolution. N. Gregory and W. S. Walker. R. & M. 3145. 1960.
Experiments were carried out in the N.P.L. 13×9 ft. wind tunnel on a 15 ft. long body of revolution of fineness ratio 10: 1. Observations were made of the effects of Reynolds number, yaw, and of isolated excrescenes on the position of transition on the solid body, and the possibilities of increasing the extent of laminar flow by means of area suction were examined. The "suction" version of the body was porous over the central third of the length and was attached to the overhead tunnel balance by means of a porous wing.—(1.1.5.1).

An approximation method for the integration of the equations of a nonstationary laminar boundary layer in an incompressible fluid. L. A. Rozin. N.A.S.A. T.T. F-22. *May* 1960.
An approximate solution is presented of the equations of a nonstationary laminar boundary layer in an incompressible fluid, based on the use of the momentum equations and a certain one-parameter family of velocity profiles.—(1.1.1.1).

Boundary layer equations and their boundary conditions in the case of motion at supersonic velocities in a moderately rarefied gas. Yu. N. Lunkin. N.A.S.A. T.T. F-28. *May* 1960.
Following Prandtl's boundary layer theory, equations are derived for a moderately rarefied gas which differ from the ordinary Prandtl equations by the presence of supplementary terms that contain higher derivatives of velocity and temperature; in the equations derived, the normal pressure gradient differs from zero and is expressed by supplementary terms. The boundary conditions are found with the aid of kinetic theory.—(1.1.1.4).

*Heat transfer and skin friction in magneto-gas dynamic channel flow. F. D. Hains. Boeing D*1-82-0047. *May* 1960.
An integral method is developed for the laminar boundary layer on the wall of a circular channel when the flow interacts with a magnetic field produced by a single loop of wire surrounding the channel. Using the previous results obtained by the method of characteristics for the inviscid external flow, numerical solutions are obtained. The skin friction and heat transfer rates are compared to the values for the case of non-magnetic interaction. Comparison is also made with measurements made in a shock tube.—(1.1.1.4 × 1.4.4 × 1.9.1).

An experimental investigation of the turbulent boundary layer in supersonic flow around unyawed cones with small heat transfer and correlations with two dimensional data. W. S. Bradfield. Convair Sc. Res. Lab. Res. Report 1. *March* 1958.—(1.1.3.4×1.9.1).

Dissociation effects upon compressible turbulent boundary layer skin friction and heat transfer. W. H. Dorrance. Convair Sc. Res. Lab. Res. Report 6. *April* 1960.—(1.1.3.4×1.9.1).

Transformations between two-dimensional and axially-symmetric laminar boundary layers including real gas effects. J. J. Shephard. Convair Sc. Res. Lab. Res. Note 19. *May* 1958.
It is shown by explicit derivation that the relations first presented by Mangler may be generalised to transform the boundary layer equations for steady axially-symmetric laminar flow of a compressible, viscous, heat conducting, real gas into the boundary layer equations for an equivalent two-dimensional flow. The transformation is exact. Real gas effects, dissociation/ionisation, and transpiration/mass transfer cooling are included. The transport properties μ and k and the specific heats are not restricted. The interpretation and applicability of the transformation is discussed and quantitative rules proposed for its range of validity.—(1.1.1.4).

Mass transfer in a turbulent boundary layer. B. M. Leadon. Convair Sc. Res. Lab. Res. Note 29. *July* 1959.—(1.1.3.1× 1.1.6.1).

The breakdown of vortices in separated flow. J. P. Jones. U.S.A.A. Report 140. *July* 1960.
The phenomenon of breakdown of the vortices above sharp-edged slender wings is described and it is suggested that it is a form of hydrodynamic instability. This possibility is investigated theoretically for an infinite vortex core in which both the axial and circumferential velocities are assumed to vary rapidly. The core receives an axi-symmetric (small) disturbance and the conditions are sought for this to grow.—(1.1.4 × 1.10.2.2 × 17.1).

COMPRESSIBLE FLOW *see also* FLUID DYNAMICS
INTERNAL FLOW
MATHEMATICS
THERMODYNAMICS
AIRCRAFT OPERATION

Effect of slight blunting of leading edge of an immersed body on the flow around it at hypersonic speeds. G. G. Chernyi. N.A.S.A. T.T. F-35. *June* 1960.
A theoretical investigation of the flow about blunt leading edge bodies, immersed in a hypersonic stream of any gas is undertaken. The study utilises piston and blast-wave theory to study the flow about such shapes as a blunt leading edge slab, a blunt-nosed cylindrical rod, a thin wedge with a blunt leading edge, and a thin blunt-nosed cone. Expressions are obtained for predicting the surface pressure distribution, the shape of the bow shock, and the drag of these bodies, and these theoretical results are compared with experimental results for several of the cases considered.—(1.2.3.2 × 1.10.1.1).

Second-order slender-body theory—axisymmetric flow. M. D. Van Dyke. N.A.S.A. T.R. R-47. 1959.
Slender body theory for subsonic and supersonic flow past bodies of revolution is extended to a second approximation. Methods are developed for handling the difficulties that arise at round ends. Comparison is made with experiment and with other theories for several simple shapes.—(1.2.1.1 × 1.2.3.1).

Analytical boundary conditions for hypersonic flow around a sphere. R. D. Linnell. Convair Sc. Res. Lab. Res. Note 4.—(1.2.3.2).

Incompressible Newtonian hypersonic flow around a sphere. R. D. Linnell. Convair Sc. Res. Lab. Res. Note 5. *June* 1957.
The inviscid flow about spheres has been determined for hypersonic free stream velocities. This present solution is based on the assumption of Newtonian flow away from stagnation point, and of constant density in the region of the stagnation point. The density is expressed in terms of the shock density ratio, which permits inclusion of real gas effects. The Newtonian flow variation of surface pressure is used to obtain preliminary results. This permits obtaining the stagnation-point surface velocity gradient in a simple manner. The result is compared with test data obtained in a wind tunnel (perfect gas conditions). A preliminary estimate of the shock separation distance along the axis of symmetry is obtained by use of a separated-variable solution of the continuity equation and the assumption of a spherical shock.—(1.2.3.2×1.4).

On the one-dimensional overtaking of a rarefaction wave by a shock wave. G. F. Bremner et al. *U.T.I.A. T.N.* 33. *Feb.* 1960.
Solutions in closed form are given for the overtaking of a rarefaction wave by a shock wave in a perfect, inviscid gas. The equations were solved on a digital computer and the results presented in graphical form.—(1.2.3.2).

Blast waves produced by exploding wire. K. Oshima. Aero. Res. Inst. Tokyo. Report 358. *July* 1960.—(1.2.3.2).

CONTROLS *see* STABILITY AND CONTROL
WINGS AND AEROFOILS
POWER PLANTS
PROPELLERS

FLUID DYNAMICS *see also* BOUNDARY LAYER
COMPRESSIBLE FLOW
TESTING AND INSTRUMENTS
THERMODYNAMICS

Surface disturbances in magnetohydrodynamics. S. I. Syrovatskiy. N.A.S.A. T.T. F-23. May 1960.
A study and explanation of characteristics of surface disturbances in magneto-hydrodynamics are presented. Unusual "transient" disturbances are described which exist during continuous transitions from one type of disturbance to another. Primary consideration is given to the problem of stability of tangential disturbances. It affords a possible explanation of stable and sharply defined streams in the form of jets, bands, etc., which are observed in the atmosphere of the sun and which are difficult to explain in terms of ordinary hydrodynamics.—(1.4.4 × 8.1).

Plane Couette flow according to the kinetic theory of gases. R. E. Street. Convair Sc. Lab. Res. Note 11. *Nov.* 1957.—(1.4).

Plane Couette flow of a conducting gas through a transverse magnetic field. B. M. Leadon. Convair Sc. Res. Lab. Res. Note 13. *Dec.* 1957.
The general equations of a plane Couette flow of an electrically conducting, variable property gas in the presence of a uniform transverse magnetic field are analysed under several sets of simplifying assumptions.—(1.4.4 × 1.9.1).

A low temperature plasma tunnel using mercury vapor and radiofrequency excitation. H. U. Eckert. Convair Sc. Res. Lab. Res. Note 18. *May* 1958.—(1.4.4 × 1.12.1 × 32.2.1).

Transport properties of free molecule (Knudsen) flow. G. N. Patterson. U.T.I.A. Review 11. *March* 1958.
The present state of knowledge of the interaction of gas molecules with the surface of a solid body as expressed by various accommodation coefficients is reviewed. The importance of the function which describes the distribution of molecular velocities is indicated for both isentropic and nonisentropic flows. Consideration is given to the determination of the aerodynamic forces and heat transfer in free molecule flow. The application of the theory to free molecule probes and the use of the equilibrium temperature probe in observing the properties of the shock transition are discussed. A wind tunnel for low density studies is also described. Some extrapolation into the realm of hypersonic flow is included.—(1.4 × 1.2.3.2 × 1.9.1).

Recent trends in the mechanics of highly rarefied gases. G. N. Patterson. U.T.I.A. Review 16. *Jan.* 1960.
A review of some recent investigations in the flow of highly rarefied gases is presented. The basic nature of the transport process in free molecule flow is deduced from the Boltzmann equation for the molecular velocity distribution function. The present semi-empirical state of our knowledge of the reflection of molecules from the surface of a solid is summarised and some directions for research are indicated. The aerodynamic properties of bodies in highly rarefied flows is considered with emphasis on the long cylinder as a case of special interest. The theory is extrapolated to the limit of very high speed ratios or Mach numbers and the results are compared with those deduced from the Newtonian flow theory. The application of recent studies of rarefied gas flows to the development of instruments for the measurement of the pressure, temperature and density of such flows is reviewed in some detail. The use of free molecule probes for the study of boundary layers and shock waves is outlined. Some factors involved in the collision-free flow of plasma are briefly considered.—(1.4 × 1.2.3.1).

INTERNAL FLOW *see also* STABILITY AND CONTROL
TESTING AND INSTRUMENTS

Design of axisymmetric exhaust nozzles by method of characteristics incorporating a variable isentropic exponent. E. C. Guentert and H. E. Neumann. N.A.S.A. T.R. R-33. 1959.
An analytical method for including thermodynamic data with variable isentropic exponent in the method of characteristics as applied to the design of high-temperature exhaust nozzles has been obtained. Several bell-shaped nozzles have been designed with this method.—(1.5.1 × 1.2.3.1 × 27.3 × 34.1).

Contribution a l'etude des diffuseurs courts de revolution. O. Aidi. Pubs. Sc. et Tech. 362. 1960. (*In French*).
The short axi-symmetric diffuser studied is formed by a rapid expansion in front of which is placed a flat plate perpendicular to the axis. Definition and measurement of efficiency, effects of the shape of the profile, of the distance away of the plate, of the velocity distribution at the entry, of a rotation, and of suction of the boundary layer, are given.—(1.5.1.4 × 1.1.5).

LOADS *see* WINGS AND AEROFOILS

PERFORMANCE *see also* STABILITY AND CONTROL
PROPELLERS

A survey of the problem of optimizing flight paths of aircraft and missiles. A. Miele. Boeing D1-82-0050. July 1960.
A survey of the problem of optimising flight paths of aircraft and missiles is presented. Quasi-steady flight over a flat Earth, non-steady flight over a flat Earth and non-steady flight over a spherical Earth are discussed.—(1.7).

STABILITY AND CONTROL *see also* WINGS AND AEROFOILS
HYDRODYNAMICS
MATHEMATICS
POWER PLANTS
PROPELLERS

Subsonic wind-tunnel tests of various forms of air intake installed in a fighter-type aircraft. B. J. Prior and C. N. Hall. R. & M. 3134. 1960.
Tests have been made of Mach numbers up to 0·93 in the R.A.E. 10 × 7 ft. high speed wind tunnel to examine how the longitudinal characteristics of fighter-type aircraft are affected by the installation of air intakes in the nose, fuselage sides, or wing roots.—(1.8.2.2 × 1.7 × 1.5.1 × 27.1).

Measurements of the direct hinge-moment derivatives at subsonic and transonic speeds for a cropped delta wing with oscillating flap. J. B. Bratt et al. *R. & M.* 3163. 1960.
Measurements of the direct hinge-movement derivatives at subsonic and transonic speeds have been made for a cropped delta wing of aspect ratio 1 : 8 with an oscillating full-span flap. These were obtained with new derivative apparatus fitted to the N.P.L. 9½ in. high speed tunnel, and some account is given of the estimation of apparatus errors. The effect of amplitude of oscillation and frequency parameter on the derivatives has been investigated.—(1.8.0.2 × 1.3.4 × 1.10.2.2).

THERMO-AERODYNAMICS *see also* BOUNDARY LAYER
FLUID DYNAMICS
THERMODYNAMICS

A review of work relevant to the study of heat transfer in hypersonic separated flows. T. Sprinks. U.S.A.A. Report 138. *June* 1960.
A review of previous work on hypersonic boundary layers and on the separation of moderately supersonic flows is presented in order to shed some light on the problem of the heat transfer in separated hypersonic flow.—(1.9.1 × 1.1.4.4).

Stagnation point heat transfer for hypersonic flow. M. F. Romig. Convair Sc. Res. Lab. Res. Note 1. *Sept.* 1956.
A design method is given for obtaining stagnation point heat transfer on a spherical nose in hypersonic flow. The heat transfer equation incorporates real gas effects through use of dissociated air properties and a fitted curve for the stagnation point velocity gradient.—(1.9.1).

Estimation of turbulent heat transfer at the sonic point of a blunt-nosed body. M. Sibulkin. Convair Sc. Res. Lab. Res. Note 7. *Oct.* 1957.—(1.9.1).

A nomograph for stagnation point heat transfer rate during hypersonic ballistic re-entry. M. F. Romig. Convair Sc. Res. Lab. Res. Note 15. *March* 1958.—(1.9.1 × 8.2 × 25.2).

A technique for experimental investigation of heat transfer from a surface in supersonic flow at large surface-to-free-stream temperature ratios. W. S. Bradfield et al. *Convair Sc. Res. Lab. Res. Note* 17. *May* 1958.—(1.9.1).

WINGS AND AEROFOILS *see also* BOUNDARY LAYER
COMPRESSIBLE FLOW
STABILITY AND CONTROL

On the relation between the generation of a lift force on a wing and the character of the flow in the boundary layer. I. V. Ostoslavskii and T. A. Grumondz. N.A.S.A. T.T. F-26. May 1960.
Results of a theoretical and experimental investigation of the mechanism involved in the generation of lift on a wing as the wing is started from rest are presented. A comparison of experiment with theory is included.—(1.10.1.1 × 1.10.2.1 × 1.1.1.1).

Aerodynamic characteristics of rectangular wings of small aspect ratio. I. J. Campbell et al. *R. & M.* 3142. 1960.
The results of a programme of low-speed wind tunnel measurements of the lift, drag and pitching moment on a number of model wings, each fitted with a full-span plain flap, and of a much more limited programme of hinge-moment measurements are presented. The wings differed from one another in profile but all were of rectangular plan form and of aspect ratio 1·25. The flap chord was varied between 0·125 and 0·500 of the overall chord. Three of the profiles investigated were conventional in having zero trailing edge thickness but the other three had large trailing edge thickness, a feature thought to be of interest in connection with the design of torpedo fins. Theoretical values of the lift slopes, hinge moments and positions of the centre of pressure were calculated from Lawrence's theory for thin rectangular wings of various aspect ratios in inviscid flow and the results are presented in tabular and graphical form. —(1.10.2.2. × 1.3.4).

*Balance and pressure measurements at high subsonic speeds on a model of a swept-wing aircraft (Hawker P.*1052) *and some comparisons with flight data. J. R. Collingbourne and A. C. S. Pindar. R. & M.* 3165. 1960.
Basic results are given of measurements in the R.A.E. 10 × 7 ft. subsonic wind tunnel over a range of lift coefficient at Mach numbers up to 0·93, the Reynolds number being 1·75 million. Though a really full analysis has not been possible, the data are discussed briefly and are compared with flight results on two aircraft of the type.—(1.10.2.2 × 1.8.0.2 × 13.2 × 1.6.1 × 3.6).

HELICOPTER AERODYNAMICS

*Jet boundary corrections for lifting rotors centered in rectangular wind tunnels. H. H. Heyson. N.A.S.A. T.R. R-*71. 1960.—(1.11.3 × 1.12.1.1).

TESTING AND INSTRUMENTS *see also* FLUID DYNAMICS
HELICOPTER AERODYNAMICS
MECHANICAL ENGINEERING
SCIENCE—GENERAL
STRUCTURES—TESTING
THERMODYNAMICS

On the choice of mercury vapor as testing medium for a plasma tunnel and some of its aerodynamic properties. H. U. Eckert. Convair Sc. Res. Lab. Res. Note 12. *Nov.* 1957.—(1.12.1 × 1.4.4 × 1.5.1.4).

Design study of the U.T.I.A. low density plasma tunnel. J. B. French and E. P. Muntz. U.T.I.A. T.N. 34. *March* 1960.
A design study of a low density plasma tunnel has been completed. A combination of an electric-arc-powered source of high energy plasma and a cryogenic pumping system, has been chosen to provide high energy flows at low pressure. The work reported covers a preliminary investigation of the arc source operating characteristics, some limit estimates of the expansion process, an experimental investigation of a " pilot plant " cryogenic pump, and the design of the overall tunnel and pumping system based on these results.—(1.12.1.3).

AEROELASTICITY

See STRUCTURES—THEORY AND ANALYSIS

AIRCRAFT

See AERODYNAMICS—WINGS AND AEROFOILS
STRUCTURES—THEORY AND ANALYSIS

AIRCRAFT OPERATION

See also AVIATION MEDICINE
POWER PLANTS
FATIGUE
STRUCTURES—THEORY AND ANALYSIS

Ground level disturbance from large aircraft flying at supersonic speeds. G. M. Lilley and J. J. Spillman. C.o.A. Note 103. *May* 1960.—(5.6 × 1.2.3.2).

*Similarity of far noise fields of jets. W. L. Howes. N.A.S.A. T.R. R-*52. 1960.
Similarity parameters for far-afield noise from subsonic and supersonic jets issuing from circular nozzles are derived and tested using experimental data. Relations for the total acoustic power, acoustic-power spectrum, acoustic directivity, local mean-square-pressure spectrum, and acoustic-pressure probability-density are considered.—(5.6).

AIRPORTS

See AVIATION MEDICINE

EXTRA-ATMOSPHERIC TECHNOLOGY

See also AERODYNAMICS—FLUID DYNAMICS
THERMO-AERODYNAMICS
METEOROLOGY
POWER PLANTS
REFERENCE LITERATURE

*Bulletin of stations for optical observations of artificial earth satellites. N.A.S.A. T.T. F-*19. *May* 1960.
Articles on optical observations of artificial earth satellites are presented which comprise the seventh of a series of publications by the Astronomical Council, Academy of Sciences, USSR, I.G.Y.—(8.2 × 24).

*The libration of a satellite. V. V. Beletskiy. N.A.S.A. T.T. F-*10. *May* 1960.
The methods of Liapunov-Chetayev are applied to find conditions for the existence and stability of the relative equilibrium of a material body in orbit in a Newtonian central-force field and to consider the oscillations of the body around the position of relative equilibrium. The problem is an idealised motion, which actually occurs in the solar system (the motion of the moon relative to the earth and the possible motions of artificial earth satellites).—(8.2).

*The problem of piercing at cosmic velocities. M. A. Lavrent'yev. N.A.S.A. T.T. F-*40. *May* 1960.
Consideration is given to the cases where a hard flat surface strikes a small cylinder and also where a hard curved surface strikes a small sphere. After the collisions, both the cylinder and sphere are assumed to vaporise.—(8.2 × 33.2.4).

Vertical re-entry of very light-weight bodies into the earth's atmosphere. R. D. Linnell. Convair Sc. Res. Lab. Res. Note 9. *Oct.* 1957.—(8.2).

AVIATION MEDICINE

The sensitivity of the precision visual glidepath (P.V.G.) at long range. J. R. Baxter et al. *A.R.L. Note H.E.* 5. *Feb.* 1960.
A series of tests were conducted to test the accuracy with which misalignments of the Precision Visual Glidepath display could be judged under a number of viewing conditions by both pilots and non-pilots. A full scale representation of the P.V.G. was observed from about 7 n. miles in hazy conditions, and a number of additional tests were conducted in a 1/1000 scale laboratory simulator using some of the same observers.—(9 × 6.5 × 5.5).

FLIGHT TESTING

See AERODYNAMICS—WINGS AND AEROFOILS

FUELS AND LUBRICANTS

See POWER PLANTS
STRUCTURES—THEORY AND ANALYSIS
THERMODYNAMICS

HYDRODYNAMICS

See also BOUNDARY LAYER

The effect of changes in the stability derivatives on the dynamic behaviour of a torpedo. J. D. Lambert. R. & M. 3143. 1960.
The extent to which the dynamic behaviour of a torpedo is sensitive to changes in its stability derivatives is investigated. The main object in carrying out the investigation was to provide guidance on the accuracy of measurement of the stability derivatives that should be necessary for any given torpedo. The considerations of the report are, however, also pertinent to the problem of deciding the effectiveness of possible changes in the design of a torpedo, the dynamic behaviour of which is unsatisfactory. Illustrative examples are worked out in detail. The importance of the so-called margin of stability is emphasised.—(17 × 1.8.0.2).

INSTRUMENTS AND EQUIPMENT

See also THERMODYNAMICS

Temperature flexure of elastic elements. G. A. Slomyanskii. N.A.S.A. T.T. F-34. *May* 1960.
A method is presented for determining the temperature flexure of an elastic element according to its experimentally determined characteristic curve, taken at a known temperature.—(18 × 21).

MATERIALS

See also INSTRUMENTS AND EQUIPMENT
FATIGUE
STRUCTURES—THEORY AND ANALYSIS

Temperature effects on material characteristics. A. J. Murphy and A. J. Kennedy. C.o.A. *Report* 135. *Aug.* 1960.
Some of the physical properties of the main elements of interest in high temperature technology are reviewed.—(21.2.2).

MATHEMATICS

The method of characteristics for routine electronic computer calculation. R. D. Linnell. Convair Sc. Res. Lab. Res. Note 3. *April* 1957.
A high speed electronic computer programme has been designed for routine calculation of supersonic flow about bodies of revolution by the method of characteristics. The equations to be solved numerically for plane and axial symmetric flow problems with or without rotation are given. The computer logic and address system necessary for general bodies are included. The body of revolution problem is used to illustrate the procedure which can be used for nozzles, jets, aerofoils and inlets.—(22.1 × 1.2.3).

A method for investigation of the properties of solutions of the equation $\ddot{x}+f(x,t)\dot{x}+g(x,t)=\phi(t)$. E. J. Putzer. Convair Sc. Res. Lab. Res. *Report* 4. *July* 1958.—(22.1).

The inertial effects of springs in resonance and free oscillation experiments to determine damping. M. D. Frost. A.C.A. *Report* 61. *Aug.* 1959.
Analyses are given of the effect of spring mass on the results of resonance and free oscillation experiments. In both cases an expression is derived for the damping coefficient and criteria are given for the reduction of each to a simple form. The theory is applied to the results of a wind-tunnel determination of the stability derivative.—(22.3 × 1.8.2.2).

On the use of spectral methods in the study of servo circuits described by stochastic differential equations of the second degree in outputs and output derivatives. M. Sundström. K.T.H. *Aero* T.N. 51. 1959.—(22.1).

MECHANICAL ENGINEERING

Machining of axially symmetric wind tunnel models. R. N. Smith. W.R. E. Australia T.N. H.S.A. 68. *June* 1960.
In an endeavour to find an optimum method of producing axially symmetric wind tunnel models, four different manufacturing techniques have been tried. The models produced have been inspected for dimensional accuracy and surface slope error, and the results compared.—(23.4 × 1.12.1).

METEOROLOGY

See also SCIENCE—GENERAL
EXTRA-ATMOSPHERIC TECHNOLOGY

Drifts and irregularities in the ionosphere. S. F. Mirkotan (Ed.) et al. N.A.S.A. T.T. F-20. *June* 1960.
Results of observations conducted at the Ashkhabad, Moscow, Tomsk and Khar'kov stations are presented. Sufficiently extensive and systematic data dealing with the study of ionospheric irregularities have been obtained mainly during 1957-1958 period. An article is included devoted to the study of ionospheric irregularities, conducted by means of radioastronomic methods at the Simeiz station, which was performed at an earlier date.—(24 × 8.1).

MISSILES

See AERODYNAMICS—THERMO-AERODYNAMICS
POWER PLANTS

POWER PLANTS

See also AERODYNAMICS—INTERNAL FLOW
STABILITY AND CONTROL
SCIENCE—GENERAL
THERMODYNAMICS

Trajectories with constant tangential thrust in central gravitational fields. W. E. Moeckel. N.A.S.A. T.R. R-53. 1960.
Trajectories followed by vehicles propelled by a constant thrust directed parallel to the velocity were calculated in non-dimensional form for a range of initial thrust-weight ratios from 10 to 10^{-4} and for a range of jet velocities from those attainable with chemical rockets to infinity. Trajectories are presented for the departure outward from, and the inward return to, any circular orbit. Results are given in the form of curves of radial distance, energy per unit mass (hyperbolic velocity), trajectory inclination, and velocity and angular distance as functions of time. Utilisation of these charts to estimate weight requirements for interplanetary missions is discussed.—(27.3 × 8.2).

Propellant vaporization as a design criterion for rocket-engine combustion chambers. R. J. Priem and M. F. Heidmann. N.A.S.A. T.N. D-252. *May* 1960.
Calculated vaporisation rates and histories for sprays of heptane, ammonia, hydrazine, oxygen, and fluorine are presented. The results are correlated with an effective chamber length for easy use for design purposes.—(27.3 × 14 × 34.1.1).

Effect of transverse acoustic oscillations on the vaporization of a liquid-fuel droplet. P. R. Wieber and W. R. Mickelsen. N.A.S.A. T.N. D-287. *May* 1960.
A theoretical analysis of the effect of a standing transverse acoustic field on the vaporisation of a liquid *n*-octane droplet was made. Three differential equations expressing the drop axial and transverse acceleration and vaporisation rate were solved on an analogue computer. Histories of the drop velocity, displacement, radius, and rate of change of radius with time were obtained.—(27.1 × 27.3 × 27.4 × 34.1.1 × 14).

Investigation of the natural frequencies of fluids in spherical and cylindrical tanks. J. L. McCarty and D. G. Stephens. N.A.S.A. T.N. D-525. *May* 1960.
Several propellant-tank configurations applicable to missile designs were oscillated to study the natural frequencies of a contained fluid over a range of tank sizes and fluid depths and to develop expressions relating these natural fluid frequencies to certain physical parameters. The configurations included spheres and circular cylinders that could be oriented with respect to the direction of oscillation.—(27.3 × 14 × 25.4.17.1 × 2 × 33.2.4.3.10).

Full-scale wind-tunnel tests of a swept-wing airplane with a cascade-type thrust reverser. M. W. Kelly et al. N.A.S.A. T.N. D-311. *April* 1960.
Results are presented of a full-scale wind tunnel investigation of an F-100F aeroplane equipped with a cascade-type thrust reverser. Longitudinal and lateral-directional stability and control data are presented for several thrust reverser configurations.—(27.1.1 × 1.8.0.2 × 1.3.4 × 1.3.6).

Practical considerations for ionic propulsion. O. E. Myers. Convair Sc. Res. Lab. Res. Note 26. *Feb.* 1959.
Ionic propulsion is restricted to applications involving extremely low thrust. It is therefore necessary that an ion rocket be given a satellite velocity by chemical means. Methods for the determination of optimum propulsion system for a specific mission are outlined and illustrated by example of calculations : 100 kg initial gross weight from a 500 km orbit to the altitude of the moon in six months, and 300 kg initial gross weight from escape velocity to the orbit of Venus in one year. The advantages of an overall system consideration are demonstrated by this treatment.—(27 × 8.2).

Optimum thrusts for rockets in vertical flight in vacuum. R. D. Linnell. Convair Sc. Res. Lab. Res. Note 34. *Feb.* 1960.
It is shown that fairly accurate optimum thrust can be calculated by including the effect of rocket-engine weight while neglecting aerodynamic drag. The results for vertical take-off with constant thrust in a constant gravity field are presented for optimisation to obtain maximum burnout velocity or maximum burnout energy.—(27.3 × 8.2).

Radial grids with water injection for suppressing the noise of jet engines. H. U. Wisniowski. N.R.C. Report L.R.-281. *March* 1960.
Noise-suppression experiments were made with water sprays introduced into the stream of exhaust gases from a jet engine by means of radial grids with tubular perforated arms.—(27.1 × 5.6).

PROPELLERS

A wind-tunnel investigation of three propellers through an angle-of-attack range from 0° *to* 85°. *P. F. Yaggy and V. L. Ragallo. N.A.S.A. T.N. D*-318. *May* 1960.
Two of the propellers were rigid and one had flapping hinges. The range of advance ratios included those anticipated for certain V.T.O.L./S.T.O.L. aircraft in take-off, transition, and landing. Measurements were obtained of power input to the propellers and the propeller forces and moments.—(29.1 × 29.7).

Investigation of the aerodynamic characteristics of a combination jet-flap and deflected-slipstream configuration at zero and low forward speeds. K. P. Spreemann and E. E. Davenport. N.A.S.A. T.N. D-363. *May* 1960.
An investigation of the aerodynamic characteristics of a semi-span wing equipped with a 38·5 per cent chord sliding flap partially immersed in the slip-stream of two propellers has been conducted. A jet of air was ejected over the knee of the flap through a momentum range from zero to about 1·0. This investigation was made to provide data on the combined effects of a jet flap and a flap-deflected slipstream at zero and low forward speeds. Pressure distributions near the wing leading edge behind the inboard propeller for conditions out of ground proximity and within ground proximity were obtained.—(29.9 × 1.3.4 × 1.7 × 1.8.2.2).

REFERENCE LITERATURE

International geophysical year bibliography of literature in the Russian language for 1958. *R. F. Zatrutina and Z. M. Mikhaylova. N.A.S.A. T.T. F*-12. *May* 1960.
The titles of 625 books and articles devoted to general and specialised questions of the I.G.Y., are contained. A continuation of the publication "International Geophysical Year—Bibliography of Literature in the Russian Language for 1954-1957".—(30.1 × 8).

Abstracts of colloquia presented at the School of Aeronautical Engineering during the academic year 1959-1960. *P. S. Lykoudis. Purdue Univ. Report Misc.*-60-3. *June* 1960.—(30).

FATIGUE

See also STRUCTURES—THEORY AND ANALYSIS

Some fatigue tests on notched specimens with programme loading for a "ground attack" aircraft. W. A. P. Fisher. C.P. 497. 1960.
A series of programme loading tests was made in a hydraulic fatigue testing machine to check the validity of the Miner Cumulative Damage Hypothesis for a structural light alloy notched specimen in axial tension.—(31.2.2.3.2.1 × 21.2.2).

Programme fatigue tests on notched bars to a gust load spectrum. W. A. P. Fisher. C.P. 498. 1960.
A series of programme fatigue tests in axial loading was made on notched bars machined from extruded alloy to D.T.D. 363A. The load spectrum represented a simplified truncated gust spectrum.—(31.2.2.3.2.1 × 21.2.2).

The fatigue notch sensitivity of annealed copper. J. Y. Mann. A.R.L. Report S.M. 272. *Sept.* 1959.
Rotating cantilever fatigue tests are reported on notched annealed copper specimens having K_t values of 1·5, 2·5, 4·0, 5·5 and >10.—(31.2.2.5.1.3 × 31.2.2.5.2.1 × 21.2.2).

On models for the probability of fatigue failure of a structure. E. Parzen. A.G.A.R.D. Report 245. *April* 1959.
Some of the problems involved in evaluating structural safety are surveyed. The probabilistic considerations involved in evaluating the strength of materials, and the construction of so called *S-N* curves, are reviewed. A probabilistic model for the life before fatigue failure of a structure is advanced.—(31.2.4.1 × 5.3 × 21.2).

SCIENCE—GENERAL

See also AERODYNAMICS—FLUID DYNAMICS
STRUCTURES—THEORY AND ANALYSIS
THERMODYNAMICS

Solar and diurnal effects in the upper atmosphere from observations of artificial earth satellites. W. Priester and H. A. Martin. R.A.E. Lib. Trans. 901. *May* 1960.
The density of the atmosphere at heights of 210, 350 and 660 km. is calculated from the orbits of satellites 1958 Delta, 1958 Alpha and 1958 Beta. A model of the atmosphere is proposed.—(32.2 × 24).

The complex conductivity of plasma of an arc discharge supported by a direct current. J. Kracik. N.A.S.A. T.T. F-24. *May* 1960.
An expression is sought for the complex conductivity of the plasma of an arc discharge supported by a strong d.c. current for the case in which a very weak high-frequency current that does not affect the condition of the plasma passes through it.—(32.2.1 × 34.1 × 27).

An experimental determination of the absolute oscillator strengths of the lines of two N.II supermultiplets. F. Mastrup. N.A.S.A. T.T. F-38. *May* 1960.
The results of measurements made on some spectral lines of nitrogen at a temperature of 20,000° are given. The apparatus for producing this temperature, a wall stabilised arc, is described.—(32.2.4).

The interactions between nitrogen and oxygen molecules. W. E. Meador. N.A.S.A. T.R. R-68. 1960.
Lippincott's delta-function model for atomic interactions is analysed, both physically and mathematically, and extended, by differentiation between K- and L-shell electrons and the introduction of a variable parameter in the expression for the delta-function strength, to cover homo-nuclear molecules more complex than hydrogen.—(32.1).

Electrical measurements on a plasma in an electrodeless discharge. J. B. Roes. Convair Sc. Res. Lab. Res. Note 23. *Nov.* 1958.
A method is developed to determine the electrical characteristics of a plasma in an electrodeless discharge system. The changes in the electrical parameters of the excitation coil are related to changes in the plasma. It is found that the electron acceleration in a high intensity discharge is limited to a layer whose thickness is related to frequency of excitation and conductivity of the plasma and should be a design parameter for discharge apparatus.—(32.2.1).

Performance of a D-C arc-excited plasma generator. W. K. McGregor et al. *A.E.D.C.-T.N.*-60-112. *Aug.* 1960.
The voltage against current relationship at different pressures is examined in the light of previous work on simple enclosed arcs. Also an energy balance is presented from which the enthalpy increase of the gas and the energy conversion efficiency is determined. Data are reported for both argon and helium used as the working gas. A study of gas contamination due to electrode erosion is presented.—(32.2.1 × 32.2.2).

Application de la méthode optique du contraste de phase a l'étude des écoulements gazeux. R. Bouyer. Pubs. Sc. et Techs. 361. 1960.
The application of optical phase contrast methods to the study of the aerodynamics of jets, the application of a usable, quantitative relationship between the density of the jet, and the photographic image of the jet, are discussed. Dioptric lenses for visualisation arrangements are discussed.—(32.2.4 × 1.12).

STRUCTURES

LOADS *see also* THEORY AND ANALYSIS

A photographic study of the impact between water drops and a surface moving at high speed. D. C. Jenkins and J. D. Booker. C.P. 501. 1960.
The normal impact between 2 mm. diameter water drops and a smooth, hard, surface moving at 1,000 ft./sec. has been studied photographically and the results discussed. The speed of the radial flow resulting from the impact has been measured and an estimate made of the corresponding pressure existing between the drop and the surface. The case of impact of drops on rough, deformable and inclined surfaces has also been briefly considered.—(33.1.2).

Small oscillations of thin resilient conical shells. E. I. Grigolyuk. N.A.S.A. T.T. F-25. May 1960.
The energy method is used to obtain the natural frequencies of vibration of truncated conical shells with large apex angles and rigid rings at each base.—(33.1.2 × 33.2.4.11.10).

Criteria for comparing the effectiveness of damping treatments. D. J. Mead. U.S.A.A. Report 125. *June* 1960.
Criteria have been deduced indicating the effectiveness of a damping treatment in attenuating bending stresses, accelerations, inertia forces and sound pressure transmitted associated with simple vibrating plates under harmonic and random excitation. Coincidence sound transmission through plates is also considered. The importance is emphasised of including in these criteria the factors by which the damping treatment increases the mass and stiffness of the system.—(33.1.2 × 33.2.4.5.10).

THEORY AND ANALYSIS *see also* EXTRA-ATMOSPHERIC TECHNOLOGY
POWER PLANTS
LOADS

Bonding of metals: important factors of influence on the bonding strength. H. Schwarz. R.A.E. Lib. Trans. 902. *May* 1960.
The principles of bonding technique are not yet sufficiently clear. Some of the most important factors are dealt with: wetting, surface pretreatment, roughness of the surface and thickness of the adhesive layer.—(33.2.4.13 × 21.7).

On Reissner's theory of the bending of plates. A. L. Goldenveizer. N.A.S.A. T.N. F-27. May 1960.
The classical theory of the bending of thin plates has been extended and improved by E. Reissner. His theory takes into account the transverse shearing of the plate, and the equations are obtained by an application of Castigliano's principle assuming a simple stress distribution through the thickness. Reissner has discovered an edge effect in plates due to transverse shearing, and the present paper shows that his theory predicts an additional edge effect of the same order of magnitude in the immediate proximity of the boundary. This paper also presents a generalisation of Reissner's theory using the same method but assuming a general law of stress distribution through the thickness.—(33.2.4.5.2).

Practical solution of plastic deformation problems in elastic-plastic range. A. Mendelson and S. S. Manson. N.A.S.A. T.R. R-28. 1959.
A practical method for solving plastic deformation problems in the elastic-plastic range is presented. The method is of successive approximations and is illustrated by four examples which include a flat plate with temperature distribution across the width, a thin shell with axial temperature distribution, a solid cylinder and a rotating disc both with radial temperature distribution.—(33.2. × 33.2.4.0.9).

Local instability of the elements of a truss-core sandwich plate. M. S. Anderson. N.A.S.A. T.R. R-30. 1959.
The charts presented give the compressive buckling coefficients for a single-truss-core sandwich plate, and cover a wide range of sandwich proportions and may be used for sandwiches with unequal faces. They apply to in-plane compressive loads acting parallel or perpendicular to the core direction or for various combinations of these loads.—(33.2.4.14.1).

Bending and compression tests of pressurized ring-stiffened cylinders. M. B. Dow and J. P. Peterson. N.A.S.A. T.N. D-360. April 1960.
The results of tests on pressurised ring-stiffened cylinders subjected to compression and bending are presented and discussed. —(33.2.4.3.7 × 33.2.4.3.1 × 33.2.4.3.2).

Effects of transient heating on the vibration frequencies of a prototype of the X-15 wing. R. P. McWithey and L. F. Vosteen. N.A.S.A. T.N. D-362. May 1960.
A prototype of the X-15 wing has been tested under simulated aerodynamic heating conditions to determine any changes in the effective stiffness.—(33.2.3.2.10 × 33.2.3.2.9 × 2 × 3.6).

Combinations of shear, compressive thermal, and compressive load stresses for the onset of permanent buckles in plates. G. W. Zender and J. B. Hall. N.A.S.A. T.N. D-384. May 1960.
A semi-empirical method of evaluating the onset of permanent buckles in plates is given. The results indicate that the compressive load stress which a plate can support at the onset of permanent buckles is reduced by thermal compressive stresses and by shear stresses. Results of the method are compared with experimental values for the onset of permanent buckles obtained from tests of square tubes of 17-7 P.H. (Condition T.H. 1050) stainless steel.—(33.2.4.5.6 × 33.2.4.5.9 × 21.2.1).

Analysis of frame-reinforced cylindrical shells. Part I—Basic theory. R. H. MacNeal and J. A. Bailie. N.A.S.A. T.N. D-400. May 1960.
The state of stress in and near a reinforced frame subjected to concentrated loads and moments and supported in a circular cylindrical shell is investigated. The theoretical approach is new and enables tables of coefficients for the calculation of loads per inch in the shell as well as loads and deflections in the frame to be evaluated using a digital computer.—(33.2.4.3).

Analysis of frame-reinforced cylindrical shells. Part II.—Discontinuities of circumferential-bending stiffness in the axial direction. R. H. MacNeal and J. A. Bailie. N.A.S.A. T.N. D-401. May 1960.
The stress distribution in, and adjacent to an externally loaded frame in a cylindrical shell is extended to include the effects of discontinuities of circumferential-bending stiffness in the axial direction.—(33.2.4.3.1).

Analysis of frame-reinforced cylindrical shells. Part III—Applications. R. H. MacNeal and J. A. Bailie. N.A.S.A. T.N. D-402. May 1960.
Tables are presented giving the loads and displacements in a flexible frame supported by a circular cylindrical shell and subjected to concentrated radial, tangential, and moment loads. Additional tables give the loads in the shell. Modifications to account for certain non-uniform properties of the structure are presented. This enables the one set of tables to be used for the solution of a wide variety of shell-frame problems, some of which have not been solved previously.—(33.2.4.3).

Range of interface thermal conductance for aircraft joints. M. E. Barzelay. N.A.S.A. T.N. D-426. May 1960.
More than 100 aluminium-alloy and high-temperature-alloy structural-joint specimens consisting of a stringer joined to a skin surface were tested under simulated aerodynamic heating conditions. Interface thermal conductance was determined from transient temperature records.—(33.2.4.6.9 × 33.2.4.13.9 × 21.2.2).

Creep bending and buckling of thin circular cylindrical shells. B. Erickson et al. *N.A.S.A. T.N. D-429. June* 1960.
A number of circular cylindrical shells manufactured of 5052-0 aluminium alloy were tested in pure bending, most at a temperature of 500°F., but a few at room temperature. Some specimens were unreinforced, some had longitudinal, some circumferential, and some longitudinal and circumferential reinforcing elements. The results are presented in tables and diagrams and a few theoretical considerations are given.—(33.2.4.3.2 × 33.2.4.3.9 × 21.2.2).

Free vibrations of a stiffened cylindrical shell. P. R. Miller. R. & M. 3154. 1960.
The first stage of an investigation of the response to random noise of a stiffened cylinder, representing an aircraft fuselage,

including an analysis of the lower modes of such a cylinder, is presented.—(33.2.4.3.10 × 33.1.2 × 32.2.3).

Analysis of deformations of viscoelastic bodies due to gravitational forces. G. Lianis. Purdue Univ. Report S-60-1. June 1960.
A theoretical analysis of deformations during storage of a thick hollow cylinder made of a viscoelastic material is presented. The cylinder is assumed fixed inside a rigid cylindrical case, resting with the cylinder axis horizontal. Stresses developed due to gravity forces set up strains which are magnified in the course of time due to the viscous nature of the material. The solution presented is fairly general so that it can be applied to any specific viscoelastic model. This analysis can be used to predict the deformations of solid propellant grains during storage.—(33.2.4.3 × 14.3).

Experimental study of the random vibrations of an aircraft structure excited by jet noise. B. L. Clarkson. U.S.A.A. Report 128. *Jan.* 1960.
Recordings have been made of the strains induced in a full scale rear fuselage test structure of the Caravelle air liner when one jet engine is running at maximum take-off thrust. The analysis has been concentrated on the strains in the centres of panels. An attempt has been made to calculate the panel resonant frequencies theoretically.—(33.2.3.3.10 × 33.3.2 × 5.6).

The failing stress of Mustang wings weakened by fatigue cracks. A. O. Payne and N. L. Squire. A.R.L. Note SM.259. Oct. 1959.
An investigation has been made into the failing stress of Mustang wings weakened by fatigue cracks.—(33.2.3.2 × 33.2.4.1.2 × 31.2.4.2 × 3.6).

Creep in aircraft structures. P. W. Kleeman. A.R.L. Report S.M. 269. *Sept.* 1959.
The following aspects of creep in aircraft structures are reviewed and discussed: the creep buckling of columns, plates and shells; the redistribution of stress under creep conditions; the estimation of total structural deformation, together with the more basic topics of: Stress-strain relations, Methods of analysis, and Material data.—(33.2.3.0.9 × 33.2.4.0.9 × 33.2.4.0.8 × 33.2.1).

Further information on the deformation of thick circular plates. B. I. Green and J. P. O. Silberstein. A.R.L. Report S.M. 271. *Sept.* 1959.
In a previous report the deflection of a heavy circular plate resting on an annulus was discussed. This note deals with the effect of introducing additional support around the circumferential edge. The solutions are again used to analyse a 74 in. optical reflector.—(33.2.4.5.3).

Zur Entwicklung der hochfesten Bindemittel für Metallverklebung vom Standpunkt der Anwendung. H. Winter and H. Meckelburg. D.F.L. Bericht 125. 1960. (*In German.*)—(33.2.4.13 × 21.3).

Ein Beitrag zur Berechnung torsionbelasteter Sperrholzrohre mit elliptischem Querschnitt ohne Längsversteifungen mit Holm und Spanten. I. Teilbericht (Vorversuche). Ein Beitrag zur Berechnung torsionbelasteter Sperrholzrohre mit kriesförmigem bzw. elliptischem Querschnitt ohne Längsversteifungen mit Spanten. H. Winter and R. Prinz. D.F.L. Bericht 131. 1959 (*In German.*)—(33.2.4.3.5).

*The reduction in stiffness after exceeding the buckling load of simply supported flat panels that change in thickness discontinuously. J. P. Benthem and J. v. d. Vooren. N.L.L.-T.N. S.*558. *March* 1960.
A method to calculate the reduction in stiffness for loads in small excess of the buckling load is applied for two cases of simply supported panels that change in thickness across the bay.—(33.2.4.5.6).

Indagine teorica e sperimentale sulle strutture composte con particolare riguardo alla resistenza a fatica. V. Villa. Mono. Sc. di Aeronautica. No. 6. *December* 1959. (*In Italian.*)
A theoretical and experimental investigation on composite structures with special regard to fatigue strength.—(33.2.4 × 31.2.3.3).

On torsional rigidity and torsional vibration of aerodynamically heated wings having a small amount of pretwist. K. Miura. Aero. Res. Inst. University Tokyo. May 1960.
An approximate analysis of torsional rigidity and of torsional vibration of aerodynamically heated wings having a small amount of pretwist is presented. The combined effects of pretwist and thermal stress on the torsional rigidity are made clear.—(33.2.3.1.5 × 33.2.3.2.5 × 33.2.3.1.9).

Thermal buckling of rectangular plates. K. Miura. Aero Res. Inst. Tokyo, Report No. 353. *May* 1960.
Thermal buckling of the plate simply supported by the web is analysed when the system is subjected to an arbitrary symmetrical temperature distribution over the plate surface. The buckling criterion is established and a simple formula for buckling criterion is obtained with reasonable accuracy for engineering purposes. Special attention is paid in exactly satisfying the boundary conditions, and the solution thus obtained is compared with those of Hayashi, Klosner and Foray, who have treated it approximately.—(33.2.4.5.9).

Symmetrical buckling of cylindrical shells under external pressure. M. Sunakawa and M. Uemura Aero. Res. Inst. University Tokyo. Report 356. *May* 1960.
The buckling phenomenon of cylindrical shells under external pressure was analysed on the basis of the theory of finite deformation. Considering the case of clamped straight edges and simply supported circumferential edges, the relations between the pressure and the deflection were obtained and the critical buckling pressures discussed.—(33.2.4.3.6 × 33.2.4.11.7).

TESTING

An improved strain-gauge type of load-cell thrust transducer. D. S. Dean. R. & M. 3153. 1960.
The development of a novel type of load cell which possesses certain advantages over the normal pillar type is described. Sensitivity is greater since the two concentrically nested tubular load-carrying members are equally loaded in compression and tension and the strains of both are indicated. This construction is more tolerant of non-axial loading, and a simple modification allows the cell to respond only to the axial component of such loads. When the normal strain of the tubes is exceeded by overload, the base of the inner tube presses on the cell casing and the load is transmitted solidly, allowing overloads of at least ten times normal full load to be carried without damage.—(33.3.1 × 1.12.6.2).

THERMODYNAMICS

See also AERODYNAMICS—INTERNAL FLOW
POWER PLANTS
SCIENCE—GENERAL

Heat conduction through a gas with one inert internal mode. J. F. Clarke. C.o.A. Note 102. *May* 1960.
The rate of energy transfer between parallel flat plates is evaluated when the (stagnant) gas between them is polyatomic with one inert internal mode. Deviations of the thermal conductivity from the complete equilibrium (Eucken) value are expressed in terms of the inert mode relaxation time and the effectiveness of the walls in exciting or de-exciting this mode. The results are obtained by a linear theory consistent with small temperature differences between the plates.—(34.3.3 × 32.1).

Theoretical and experimental investigation of heat conduction in air, including effects of oxygen dissociation. C. F. Hansen et al. *N.A.S.A. T.R. R-*27. 1959.
Solutions are presented for the conduction of heat through a semi-infinite gas medium having a uniform initial temperature and a constant boundary temperature. The coefficients of thermal conductivity and diffusivity are treated as variables, and the solutions are extended to the case of air at temperatures where oxygen dissociation occurs. These solutions are used together with shock-tube measurements to evaluate the integral of thermal conductivity for air as a function of temperature.—(34.3.3 × 1.9.1).

*Temperature and composition of a plasma obtained by seeding a cyanogen-oxygen flame with cesium. R. A. Hord and J. B. Pennington. N.A.S.A. T.N. D-*380. *May* 1960.
The temperature and composition of a cyanogen-oxygen flame seeded with caesium are calculated approximately by assuming that the combustion is adiabatic and that the product mixture is in thermodynamic equilibrium.—(34.1 × 1.4 × 27).

Mollier diagram for nitrogen. R. L. Humphrey et al. *A.E.D.C.-T.N.*-60-83. *May* 1960.
Thermodynamic data for nitrogen are presented in Mollier diagram form covering the temperature range of 30 to 15,000°K and the density range of 10^{-6} to 10^{3} atmospheres.—(34.1 × 32.2.2 × 1.9).

Non-equilibrium centered rarefaction for a reacting mixture. L. G. Napolitano. A.E.D.C.-T.N.-60-129. *June* 1960.
General equations are derived which describe the dynamics and thermodynamics of an ideal dissociating gas when only one relaxation time, the chemical one, is different from zero. The equations are applied to the study of an expansion around a two-dimensional corner by the methods of the characteristics.—(34.4 × 32.1 × 1.2.3.1).

Kinetics of gases. Gas Dynamics Lab. Northwestern University. A.E.D.C.-T.N.-60-130. *July* 1960.
The fundamental features of real gas effects are reviewed with emphasis on high temperature effects. Topics considered include the thermal and caloric equations of state, basic effects in flow processes, and the speed of sound in reacting gases. The kinetics of the processes of dissociation and recombination and ionisation and charge neutralisation are discussed with some applications. Analyses are presented describing chemical non-equilibrium effects in some flow situations, among them inviscid flow after sudden changes in fluid properties and the mixing of parallel streams.—(34.4 × 1.9).

Dissociation and ionization effects in the laminar mixing of parallel streams. R. F. Hoglund. A.E.D.C.-T.N.-60-141. *July* 1960.
The boundary layer approximation is applied to the equations governing the mixing zone between two parallel semi-infinite streams. The two streams differ in composition and temperature but they are not necessarily in chemical equilibrium. Closed form solutions are obtained for the approach to equilibrium in the mixing zone in the case of equal velocities of the two streams. Frozen flow and equilibrium flow appear as limiting cases of the general non-equilibrium situations.—(34.4 × 1.9.2 × 1.1.1.4).

On near-equilibrium and near-frozen behavior of one-dimensional flow. M. H. Bloom and L. Ting. A.E.D.C.-T.N.-60-156. *July* 1960.
Near-equilibrium and near-frozen one-dimensional inviscid flows downstream of an initial station at which the flow is in equilibrium have been examined by means of perturbation analyses. The near-equilibrium analysis requires the application of boundary layer techniques. The near-frozen analysis is of standard type. Representative numerical results are included.—(34.4).

Flame reaction rates. A. L. Berlad. Convair Sc. Res. Lab. Res. Note 10. *July* 1957.—(34.1.2).

On non-adiabatic flame propagation. A. L. Berlad et al. *Convair Sc. Res. Lab. Res. Note* 20. *July* 1958.—(34.1.2).

On the existence of steady state flames. A. L. Berlad and C. H. Yang. Convair Sc. Res. Lab. Res. Note 21. *July* 1958.
Examination of both adiabatic and non-adiabatic flame theories shows that a "steady state" exists only under the special condition that a heat sink exists at the initial temperature. For the general case of freely propagating, non-adiabatic flames only a quasi-steady state can be achieved.—(34.1.2).

Spark ignition of a fuel spray. K. Iinuma. Tech. Coll. Hosei University, Report 7. *March* 1960.
The results of a series of experiments with a single spark are described, in which the effects of timing and location of the spark upon the ignitibility are examined for several types of fuels. Ignitibility of a fuel spray by repeated spark is treated, and the experimental results are interpreted on the basis of probability considerations.—(34.1.1 × 14.1).

A heat-flux-meter for use with thin film surface thermometers. R. F. Meyer. N.R.C. Report L.R-279. April 1960.
Thin film surface thermometers may be used to measure the heat transfer to a body on which they are mounted if the assumption of one-dimensional heat transfer to the body is made. This, used with the thin film thermometer, enables the heat transfer to be recorded directly.—(34.2.1 × 1.12.1 × 18.1).

Relation entre la structure chimique et la réactivité des combustibles pour fusées a l'acide azotique constantes physiques des combustibles. J. Francon. Pubs Sc. et Tech. T.N. 89. 1960. (*In French*).
An attempt to find a relationship between the chemical structure of a rocket fuel and its reaction with NO_3H 48°, is made. Speed of combustion and some of the constants of combustible liquids are given.—(34.1.1 × 27.3).

Ignition and flame stabilization of stream of combustible gaseous mixtures by hot jet. H. Tsuji. Aero. Res. Inst. Tokyo Report 357. *June* 1960.
The problem of the ignition of the combustible gaseous stream by a hot jet is formulated and analysed, considering a simplified model and using the boundary layer approximation of aerothermochemistry. A one-step unopposed spherical reaction following any order reaction kinetics with temperature dependence according to the Arrhenius rate lay is assumed. The Prandtl and Schmidt numbers are assumed to be equal, then a similarity between the equation of energy and the diffusion equation exists, greatly simplifying the analysis. Approximate but general analytic expressions for the N^{th} approximation to the temperature and concentration profiles are obtained. These analytic results are applied to the problems of the thermal decomposition of azomethane and the thermal decomposition of hydrazine. Numerical calculations are carried out for the first-order approximation.—(34.1.1).